A NEW APPROXIMATE ANALYTICAL SOLUTION FOR A HEAT TRANSFER PROBLEM IN A TROMBE WALL

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ABSTRACT

The present investigation is a theoretical one. A new technique is devised to predict both the temperature and velocity distributions in the Trombe wall. A new analytical solution is found for the two dimensional boundary layer equations in a Trombe wall channel for steady state regime with a constant wall temperature. The unsteady state case is worked out and an analytical solution is found for temperature, nondimensional stream function, and Blasius velocity profiles. An analytical solution is found for the case when the wall temperature is not uniform but of the form of the power law distribution. The analytical results are compared with those obtained using different numerical techniques and with some available experimental data. A good agreement is found between these results.

NOMENCLATURE

C_{1-15}	constants
C_p	specific heat at constant pressure
d_{1-8}	constants
D	substantial derivative
f	nondimensional stream function
G	generalized Grashof number
Gr	grashof number
g	gravity acceleration
k	thermal conductivity
L	characteristic length
M_{1-3}	constants
Nu	Nusselt number
P	pressure
Pr	Prandtl number
Q	generated heat
Re	Reynold number
T	temperature
T_w	wall temperature
T_a	ambient temperature
u	x-component velocity
V	velocity
v	y-component velocity
x	vertical coordinate,see Fig.(3.1)
y	coordinate normal to the wall.Fig.(3.1)

Greak symbols

δ	constant
ϵ	constant
ν	kinematic viscosity
μ	dynamic viscosity
ρ	fluid density
β	volumetric thermal expansion coefficient
α	thermal diffusivity.
θ	dimensionless temperature
ψ	stream function
η	similarity variable
τ	time

Subscripts

a	air
p	particular
c	complementary
w	wall

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Chapter 1

INTRODUCTION

1.1 Introduction

With the ever growing concern for energy conservation, a great deal of interest has recently been shown in the use of solar energy for the purpose of heating dwellings.

Since the construction in 1967 in France of the first house with a "Trombe wall" there has been continuing interest regarding the potential of passive solar systems. A Trombe wall Fig.(1.1.a) is essentially high capacitance solar collector coupled directly to the spaces to be heated.

Solar radiation is absorbed on the outer surface of the wall. Energy is transferred from the room side of the wall to the spaces to be heated, by convection and radiation. Energy can be transferred to the room by air circulating through the gap between the wall and glazing through openings at the top and bottom of the wall. The surface of the mass wall facing south (in the northern hemisphere) is blackened and glazed. The surface gets heated by the sun during the shine hours and may be covered by insulation during the offshine hours to reduce heat losses to outside air. The thermal storage wall may be made of concrete, or water, it is common to stackup drums full of water, one above the others.

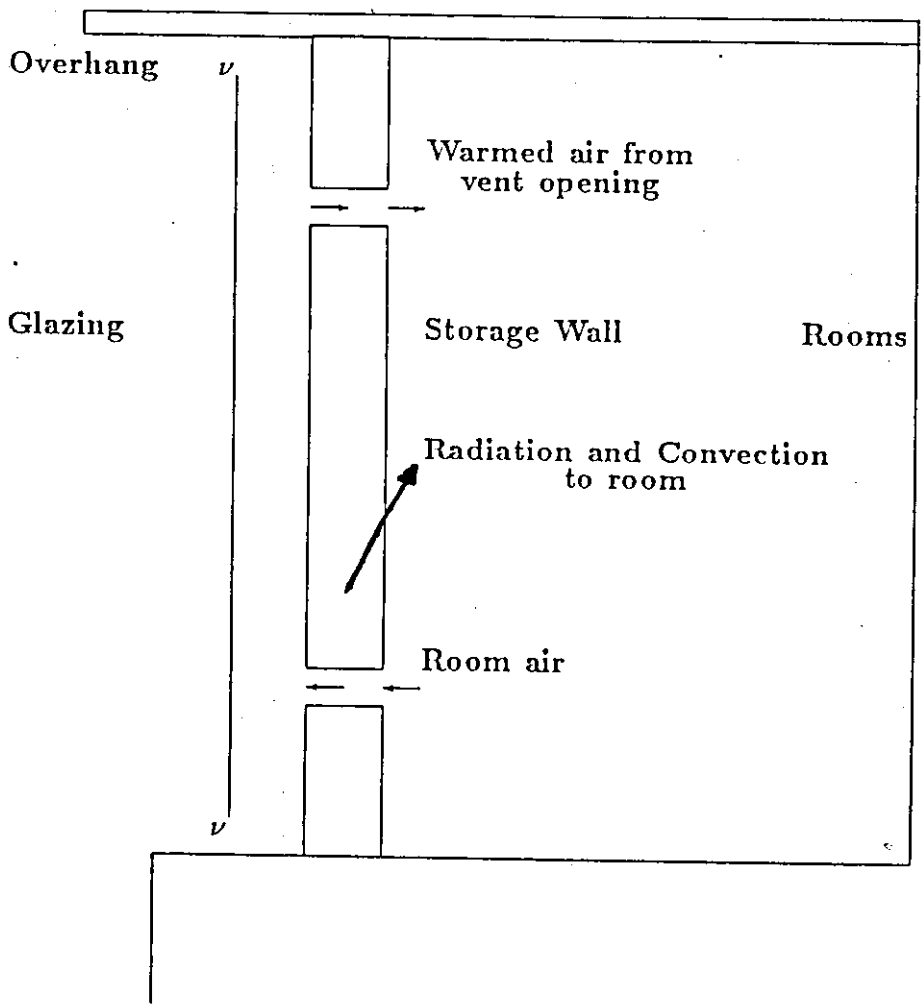

Fig.1.1.a. vent dampers in Trombe wall are used to vent the gap between glazing and wall in summer.

1.1.1 Concrete Trombe Wall

Trombe wall is essentially a thick wall, with the outer surface blackened and glazed. The storage mass is concrete. Solar radiation is absorbed by the blackened surface and is stored as sensible heat in the wall as shown in Fig.(1.1.b).

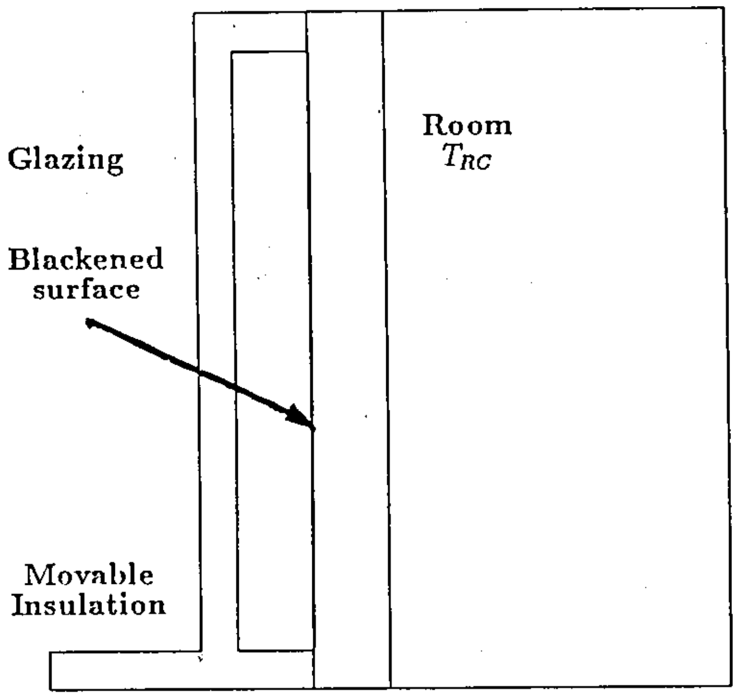

Fig.1.1.b Concrete Trombe Wall

1.1.2 Water Trombe Wall

Water wall is based on the same consideration as concrete Trombe wall except that it employs water as the storage material. It consists of containers (metallic) filled with water and is kept south facing. One surface of the wall is blackened and glazed while the other surface can either be in direct contact with the living space or be separated from it by a thin concrete wall or insulating layer as shown in Fig.(1.1.c).

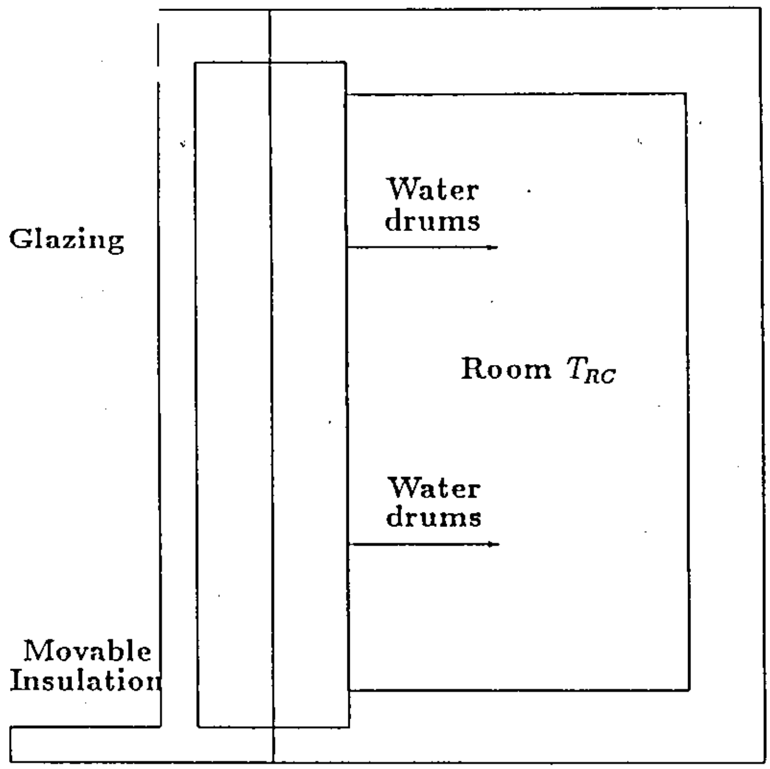

Fig.1.1.c Water Trombe Wall

1.2 Outline of Theoretical Contribution

The idea of utilizing and developing Trombe wall has been investigated by many researchers during the last twenty years. Many methods have been devised to predict and measure the temperature profile through the wall under different wall conditions.

In this study a new analytical method is devised to predict both the temperature and velocity distributions in the Trombe wall. The obtained analytical results will be compared with same numerically predicted published data, as well as same with available experimental data.

1.3 Layout of the thesis

The thesis is divided into five chapters, of which this introduction is the first. Chapter two is a literature survey of the work done on numerical and analytical prediction for laminar convection within the Trombe wall channel. Chapter 3 describes physical model and mathematical analysis of the problem.

Chapter 4 discusses the predicted results and compares them with the numerical ones. Finally chapter 5 reports the concluding remarks gained from the present work, followed by recommendations for future work.

Chapter 2

LITERATURE SURVEY

2.1 Introduction

Heat transfer by natural convection is frequently encountered in our environment and in many engineering applications. Natural convection arises from the buoyancy force induced by density differences in a fluid. Laminar free convection along horizontal, inclined and vertical plates with uniform surface temperature or uniform heat flux has been extensively studied.

Rapidly growing acceptance of solar energy as the means of heating and cooling has further stimulated research in the area of thermo-gravitational flows in open ended cavities and parallel wall channel configuration that simulates passive solar systems such as the Trombe wall. Nayak [1] compared the performance of two types of thermal storage wall, namely Transwall and Trombe wall. Both of them were located directly behind the south facing glazing of a wall.

Most of the previous work related to Trombe wall channel flow can be grouped into two major categories; the first involves numerical solution for the governing equations to predict the temperature and velocity profiles with isothermal and constant wall heat flux conditions.

The second one is involving the analytical solutions of the governing equations using the boundary conditions available to predict the flow rate and heat transfer rate.

There are also experimental investigations on natural convection from vertical, inclined, and horizontal surfaces, covering both laminar and turbulent regimes under either a constant surface heat flux condition. However, these analytical and experimental studies were conducted under the situations of uniform thermal boundary conditions.

The aim of this chapter is to survey some of the important works carried out to predict the hydrodynamic and thermal characteristics of convection between parallel plates with more emphasis on those related to Trombe wall.

2.2 Numerical Studies

One might say that for most natural convection flow of practical interest, a complete analytical solution is not available and one has to depend on various numerical techniques to obtain the desired results. Over the past few years, with the increased availability of fast computers, very large number of numerical techniques and procedures have been developed and employed for various diverse natural convection problems.

Several numerical methods have been carried out to study the laminar natural convection and mixed natural and forced convection in a vertical channel with various wall conditions. Akbari and Borgers [2] investigated free laminar heat transfer between the channel surfaces of Trombe wall. They considered in their study the velocity profiles normal and parallel to the direction of fluid flow, the pressure drop due to flow acceleration at the channel entrance, and the effect of dissimilar but uniform channel surface temperature for a wide range of flow rates and temperatures. A finite difference procedure was used to solve the governing equations in dimensionless form using air as the working fluid. After comparison with available experimental data , results have been reduced and several correlations were developed to enable important performance characteristics to be estimated given the channel thickness height, and surface temperature.

A numerical prediction for turbulent free convection from vertical surfaces was studied by Mason and Seban [3]. They showed that the results can be obtained by means of suitable modifications of a program of the

Patankar-Spalding procedure for predicting the results. The related results compared well with the experimental ones.

An analysis was performed by Chen [4] to study the flow and heat transfer characteristics of laminar free convection in boundary layer flows from horizontal, inclined, and vertical flat plates in which the wall temperature $T_w(x)$ or the surface heat flux $q_w(x)$ varies as the power of the axial coordinate in the form:

$$T_w(x) = T_a + Nx^n$$

or:

$$q_w(x) = bx^m$$

The governing equations were first cast into a dimensionless form by a non-similar transformation and the resulting equations were solved by a finite difference scheme. They also found that, both local wall shear stress and the local surface heat transfer rate increased with the increase of both the angle of inclination from the horizontal increased and with the local Grashof number. New correlation equations for the local and average Nusselt number were obtained for the special cases of uniform wall temperature and uniform heat flux. The results were compared with available experimental data.

Borgers and Akbari [5] studied the free convective turbulent flow within the Trombe wall channel. They assumed that, the initial flow remains laminar until a combination of geometry, temperature, and flow rate conditions reach a pre-defined level. Also they predicted the turbulent flow characteristics by a mixing length model which incorporates empirical parameters used in the literature. The equations were solved using a forward marching line by line implicit finite difference technique permitting iterations on each new line.

The two dimensional, steady, combined forced and natural convection in a vertical channel was investigated for the laminar regime by Chaturredi et al [6]. They used both a finite difference method using upwind differencing for the nonlinear convective terms, and central differencing for the second order derivative to solve the governing differential equations for the mass, momentum, and energy balances. The solution was obtained for stream function, vorticity, and temperature as the dependent variables by an iterative technique known as successive substitution with over-relaxation (SOR). Chaturredi et al showed that at any low Reynolds Number, the stream function, and isotherms were qualitatively similar to those reported for the natural convection in rectangular slots.

Jubran et al [7] investigated the convective laminar heat transfer between the channel surfaces of a Trombe wall. Velocity profiles, temperature profiles, and pressure defect had been investigated when the temperature of the masonry wall is not uniform but of the form

$$T_w(x) = T_g + ax^n$$

They concluded that the average Nusselt Number increases as the rate of heat gained by the fluid at the exit was increased, i.e as n was increased.

The governing equations were first transformed into dimensionless form and then solved using the finite difference technique. The governing equations were also solved for different values of n, and hence, for different temperature distributions at the wall.

Abd Rabbo and Adam [8] developed a mathematical model based upon the solution of heat transfer equations by finite differences method under unsteady state heat flow to optimize the performance of Trombe wall under Mosul weather conditions. The analysis was carried out for south facing walls and the effect of various materials and thicknesses was studied. The thermal behaviour of conventional wall was compared with Trombe wall under the same mean environmental temperature and global solar radiation for an average day in January for Iraqi winter season. Abd Rabbo and Adam showed that the wall of 20cm thickness was the most suitable in winter and summer, and the temperature of the inside surface for Trombe wall was 17 % higher than that for a conventional wall.

Shai and Barnea [9] conducted an analysis of mixed convection with uniform heat flux. They evaluated the heat transfer coefficient in an assisting mixed convection, and in an apposing mixed convection. The analysis assumed that the hydrodynamic and thermal boundary layers were the same, and the velocity profile within the boundary layer was a superposition of pure forced and pure natural convection.

A numerical investigation was made of laminar mixed convection of air in vertical channel by Habchi and Acharya [10]. The thermal boundary condition considered was symmetric heating, where both plates were heated and asymmetric heating, where one plate was heated and the other one was adiabatic. Habchi and Acharya indicated that the Nusselt Number attained its maximum value near the inlet of the channel and increased with decreasing Gr/Re^2 values. The differential equations were solved by using a Patankar-Spalding type of finite difference procedure. This implicit finite difference scheme was a marching method that begins at the channel inlet and proceeded step by step until the channel exit was reached.

Badr [11] considered a theoretical study of laminar mixed convection for horizontal cylinder in a cross stream. Badr's study was based on the solution of the Navier Stokes and energy equations for two dimensional flow of a Boussinesq fluid. Badr obtained in his work the steady solution of the governing equations through studying the time development of the velocity and thermal boundary layers around the cylinder starting from certain initial conditions. This was achieved through integrating the governing equations with time until reaching the fully velocity and temperature fields. The method of series truncation developed by Badr and Dennis [12] was applied to study the asymmetrical flow field around a rotating cylinder was adopted for tackling this problem.

Habchi and Acharya [13] investigated the laminar mixed convection of air in a vertical channel containing a partial rectangular blockage on one of the channel wall. Their results indicated that at low values of Gr/Re^2 the maximum velocity occurs near the wall, and the Nusselt Number in the blockage and the pre-blockage regions increased with decreasing Gr/Re^2 values. The differential equations were solved using an implicit, elliptic finite difference procedure called SIMPLER (Semi Implicit Method for Pressure Linked Equations, Revised). In this method the domain was subdivided into a number of control volumes, each associated with a grid point, and the governing differential equation was integrated over each control volume resulting in a system of algebraic equations that can be solved by an iterative technique. They used an approximate exponential profile to perform the integration in each coordinate direction.

Kuiken [14] considered a class of backward free convective boundary layer similarity solutions. Kuiken showed that these boundary layers can be produced along slender downward projecting slabs of prescribed thickness radiation, which were infinitely long. It was pointed out in this study that these solutions can be used to describe free convective flows along vertical fins.

A numerical solution scheme for local non-similarity boundary layer analysis was developed by Minkowycz and Sparrow [15]. In this method the central task was the numerical solution of a set of simultaneous ordinary differential equations. The numerical solution scheme described in this study was able to deal with the multi-equation system encountered in local non-similarity boundary layer analysis. It employed integrated forms of the governing differential equations. They selected a natural convection problem on an isothermal vertical plate in the presence of surface mass transfer to illustrate their solution method.

Sparrow and Gregg [16] investigated the role of buoyancy force induced by the density gradient in the combined forced and natural convection flow problems. They established a quantitative criterion for determining the situations in which the buoyancy force may be ignored. The isotherms and streamline patterns of flow produced by the interaction of buoyancy and shear forces in a vertical slot across which a constant temperature difference is maintained was determined by Elder [17]. Deval Davis et al [18] obtained the numerical solution of the governing equations subject to the Boussinesq approximation.

An analysis of convective heat transfer between vertical plates with one plate isothermally heated and the other insulated was performed by Miyatake and Fuji [19]. They used a forward marching implicit method to solve the nonlinear partial differential equations. Bodia and Ostterle [20] investigated the development of natural convection in a fluid between heated vertical plates for the conditions where the wall is not sufficiently high, and the flow is not fully developed at the exit. Chung and Thompson [21] and Ramakrishman [22] have solved the governing Navier Stokes equations for similar problem using the finite element technique.

Hasan and Eichorn [23]analyzed the effect of the angle of inclination on free convection flow and heat transfer from an isothermal surface by the local non-similarity method of solution. Numerical solutions of the equations were obtained for Prandtl Numbers of 0.1, 0.7, 6.0 and 275. Results showed an appreciable effect of inclination parameter on the velocity field and practically none on the temperature field except for very small values of the Prandtl Numbers. In the limiting case of very large Prandtl Number, inclination parameter has no effect either on the velocity or on the temperature field.

The effects of buoyancy on upward-flow laminar convection in the entrance region between inclined parallel plates were studied numerically by

Naito and Nagano [24]. Three thermal conditions of parallel plates were considered: lower wall heated and upper wall insulated; vise versa; and both walls heated equally. Solutions for these three cases were obtained. They showed that the developing upward-flow and thermal field in the entry region were affected by buoyancy in terms of the various angles of parallel plates. They developed correlations for skin-friction coefficient and local Nusselt number at an arbitrary channel inclination. The dimensionless governing equations of continuity, momentum, and energy were simplified using the usual Boussinesq approximation and then approximated by finite difference equations using a central difference form.

Raithby et al [25] presented an analysis which predicts the heat transfer across fluid layers bounded laterally by vertical isothermal surface and adiabatic surfaces on the top and the bottom. They predicted the vertical temperature distribution in the core of the cavity. Also they compared average Nusselt number and temperature distribution with experimental data for aspect ratios greater than 5 and a good agreement between analysis and experiment was found.

Faiman et al [26] developed a numerical model which describes the temperature distribution as a function of time within the storage elements of a Rotating Prism Solar storage wall. The model was tested against experimental data obtained from a full-scale Rotating Prism wall, and found to reproduce the measured temperature extensively well. The model was used to predict the performance of such devices in various kinds of climate and to compare this performance with that expected from a non-vented Trombe wall of standard design. The Rotating Prism wall was found to provide considerably more useful energy than a non-vented Trombe wall.

A computer simulation analysis of possible system designs was employed by Balcomb et al [27] to aid in the selection of components of the wall. Their results indicated that a performance comparable to that of a conventional active solar heating system should be achievable in an optimized design of passive solar heating. They showed that movable insulation of the window increased the performance when used in conjunction with a conventional heating system, temperature variations in the building can be reduced to those normally experienced.

Sebald [28] discussed in detail mathematical aspects of thermal network (TN) models. Because of its efficiency in large controlled TN models emphasis was given to forward differencing (FD). He analyzed computational considerations and accuracy. FD in TN models was shown to be accurate and efficient in complex buildings. Sebald showed that structural information contained in TN models was easily extracted as a bonus without simulation.

2.3 Analytical Studies

The solution of natural convection and mixed and forced convection in a Trombe wall channel using analytical means has received little attention in the past.

Transient response of the Trombe wall applicable to passive solar heating systems has been analyzed by Shou-Shing and Jinn-Tsong [29]. The unsteady temperature distribution and heat loss history of a Trombe wall, applicable to passive solar heating, were obtained from both analytical and numerical approaches. A one dimensional exact solution was obtained by the Duhamel superposition technique for the time dependent boundary. The results were compared with numerical ones calculated through an implicit finite difference technique.

Yao [30] investigated free and forced convection in the entry region of a heated vertical channel. He studied the conditions of constant wall temperature and constant wall heat flux. Different axial length scales were revealed by the analytical solution. The solution indicated that natural convection eventually becomes the dominant heat transfer mode if $Gr > Re$ for constant wall temperature and $Gr^2 > Re$ for constant wall heat flux. Local Nusselt Numbers had been successfully correlated by the length scale deduced from the analytical solution. The solution of the inviscid flow was obtained by expanding the dependent variables in asymptotic series. The resulting equations were numerically integrated by using fast Fourier Transformation.

An analytical exact solution using Laplace Transformation to Graetz problem for laminar fluids in circular ducts was presented by Mansour [31]. The results obtained from this solution were compared with previously published experimental, analytical and numerical works.

Some unsteady heat transfer problems solved by a simple similarity transformation were examined by Ruckenstein [32]. He used a similarity transformation for solving equations of the form:

$$\frac{\partial T}{\partial \tau} + u(x,\tau)\frac{\partial T}{\partial x} - y\frac{\partial u}{\partial x}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Several unsteady heat transfer problems have been solved using the potential flow velocity.

Nayak et al [33] considered the analysis of passive heating concepts. They developed a mathematical model based on Fourier series solution of the heat conduction equation to analyze the thermal performance of some typical passive heating concepts, namely Trombe wall, water wall and solarium. Using this model they obtained an analytical expression for the time dependent heat flux entering into the living space, which was assumed to be at a constant temperature corresponding to an air conditional room.

Chen and Tzuoo [34] studied the vortex instability characteristics of laminar free convection flow over horizontal and inclined isothermal surfaces analytically by linear theory. As a prelude to their analysis, the effect of the angle of inclination on the main flow and thermal fields were re-examined by a new approach. They found that as the angle of inclination increases the rate of surface heat transfer increases, whereas the susceptibility of the flow to the vortex mode of instability decreases. The system of equations for the main flow and thermal fields was solved by a finite difference scheme. The stability problem was solved by a fourth order Runge Kutta numerical integration scheme. They compared their results with available wave instability known results.

Chellappa and Singh [35] presented an analysis for deriving in a systematic way all possible similarity formulations for linearly viscous, free convection boundary layer flow over a semi infinite horizontal plate. These formulations were derived on the basis of the constraints imposed by similarity conditions of the problem. They concluded that there are in all four possible cases of similarity formulations for this boundary layer problem. Two of these possible cases deal with unsteady flow conditions; and the other two steady cases deal with more general forms for plate temperature distributions.

Lu et al [36] derived theoretical equations for calculating the absorbed solar radiation of a direct-gain passively-heated system with transparent glazing by using the net radiation method. A more rational model for simulating the thermal processes of a direct-gain system was worked out. The test cell validation was done and it showed good agreement between the predicted and measured values.

Zrikemand Bilgen [37] studied theoretically the transfer of solar radiation in a composite Trombe wall solar collector system. The composite system consists of a glazing, a massive wall, and an insulating wall put in contact with massive wall and with another glazing between massive and insulating walls. The theoretical results indicated that the composite system can indeed perform better in cold and/or cloudy climates than the referencc system and that optimum geometrical parameters could be determined depending on the dwelling type and climatic conditions of the area. They concluded that the new system had a reduced massive wall thickness that was a desirable feature for light-weight constructions.

Mansour et al [38] found out an approximate analytical solution to convective heat transfer flow. The results obtained in that solution showed a good agreement compared with the numerical and experimental published data. This work differs from the present work in the form of the guessed function and in using the hypergeometric functions for their solutions.

2.4 Experimental Studies

There are a few works that deal with experimental studies on natural convection, mixed natural and forced convection in a Trombe wall channel.

An experimental investigation on turbulent natural convection boundary layers has been conducted with water on a vertical plate of constant heat flux by Vliet and Lu [39]. They presented local heat transfer data for laminar, transition and turbulent natural convection with the emphasis on the turbulent basis.

Sparrow and Bahram [40] investigated experimentally the natural convection heat transfer from face to face surface of parallel, square vertical plates. The experiments encompassed three types of hydrodynamic conditions along the lateral edge: (1) fully open to ambient; (2) blockage of one of the edge gaps; (3) blockage along both of the edge gaps. Measurements, were made for the ten inter plate spacings. They used a mass transfer measurement technique instead of radiation effects, and concluded that the present data for the single vertical plate are in agreement with prior experiments and with other published correlations.

Anezov [41] developed a procedure which made it possible to compare values of the efficiency of a solar heater and the replacement factor of various heating systems in their design stage to find out the effect of one or another parameter separately on the system's overall indices and then to optimize the system with respect to all the parameters. He studied particular cases: a solar-heating system (insulation heating) and a traditional Trombe-Michel passive system. In this system there are no heating devices inside the room, and the heat-transfer medium circulates between the solar heater and the space being heated by means of natural convection.

The performance of active and passive rockbins were compared in Albuquerque, NM, Santa Maria, CA, and Madison, WI by Sebald and Vered [42]. They assumed the basic house to contain both Trombe wall and direct gain which in turn were assumed to be optimally sized and controlled for each weather season. It was demonstrated that, provided charging was done from the Trombe wall, rockbins could be used to advantage in reducing the early morning auxiliary energy consumption peak common to passive houses with night setback thermostats. They analyzed the performance sensitivity to rockbin configuration and to control strategies for charging and discharging. They included also the effects of fan energy.

Casperson [43] presented an experimental data on the thermo-circulation in a variable geometry Trombe wall for a full-scale test facility located in Idaho Springs, Colorado. Anemometry data representing velocity and temperature profiles in the Trombe wall air gap were obtained over a five month period from Oct. to March of the (1980-1981) heating season. They collected temperature and solar radiation data. These data sets completely described an experimental control volume on the east-west vertical center-plane of the Trombe wall . They collected data for four different wall air gap widths in combination with three different inlet/outlet vent cross-sectional flow areas. Wall gap widths of 2, 4, 6 and 8 inches where investigated. They collected additional thermocouples data in the air and in the inlet and outlet vents to the Trombe wall.

An experimental investigation was carried out on the hydrodynamics of turbulent free-convection boundary layer at a vertical film of ethyl alcohol closed at both ends by Kutateladze et al [44]. They presented longitudinal component profiles of the mean velocity vector and fluctuation rate of the same velocity component. The existence was shown of a quasi-stationary wall layer in the turbulent boundary layer at a vertical plate on the basis of the present data and experimental results of other authors, also the thickness and maximum velocity which satisfy the condition

$$Re_1 Pr^{1/2} = (U_{max}\delta_1)/((\nu a)^{1/2}.$$

It was found experimentally that the temperature gradient $\frac{dT}{dx} = \mathrm{const}$ along the middle of the vertical cross-section in the central part of the film.

Experimental results of a lattice solar wall were described by Yan et al [45]. Comparison tests had shown that the lattice solar wall performed better than the traditional solar wall. The lattice solar wall was aesthetically pleasing, and offered a saving in construction materials. It was shown that its superior thermal properties were due to the following: a high-heat retention capability in the course of solar radiation; the weak radiative heat transfer because of the losses; and low reflective losses on incident sunlight.

The operating characteristics of a Trombe wall collection-storage solar system were measured for conditioning validation air for a furrowing house by Siebenmorgen et al [46]. Two types of planner reflector along with the condition of no reflectors were alternately placed horizontally in front of the vertical collector. They used a linear regression analysis to determine the best set of independent climatic and system variables to describe the daily heat gain of the system for each reflector use. They found that using long-term records of solar radiation data in conjunction with the regression analysis, pay-back periods of 7 to 8 years were attainable with this system.

Yin et al [47] described an experimental investigation concerning the natural convection flow patterns which occur in the annular space between two concentric isothermal spheres, with the inner one being hotter. The diameter ratios ranged from 1.09 to 2.17. The convicting fluids were air and water, with Grashof numbers in the range of 1.7×10^3 to 1.5×10^7. They observed that the several types of flow patterns were correlated with previously published temperature profiles and were categorized in terms of steady and unsteady regimes.

Arnold et al [48] carried out an experimental investigation of steady natural convection heat transfer for finite rectangular regions. They measured the effect of angle of inclination on heat transfer across rectangular regions of several aspects ratios for Rayliegh numbers between 10^3 and 10^6. They examined the situations for which the angle of inclination varied from 0 deg. (heated from above) to 180 deg. (heated from below) with aspect ratios of one, three, six, and twelve. They compared their results with past theoretical works and a simple scaling law derived which is valid for angles of inclination from 0 to 90 deg. (vertical).

Maldonado et al [49] adapted traditional Portugues building forms to create a relatively massive two-storage house in Northern Portugal, which incorporated direct solar gain, Trombe wall and water wall systems. They discussed two year results and they concluded that exiting simulation methods under-predict the performance of massive structure.

Sebald et al [50] analyzed Trombe wall performance for a variety of control strategies in Albuquerque, New Mexico, Santa Maria, California and Madison Wisconsin. Both the presence and absence of backup energy were considered. They performed the analysis using hourly simulations on Solmet weather data in a thermal network model. They computed the sensitivity of the results to wall thickness and size, building azimuth and house insulation levels. They found that proper controls to reduce backup requirements as much as 50%. In the absence of backup energy, proper controls on thin walls provided better performance than standard walls of double thickness.

An annual simplified method has been developed by Zrikem and Bilgen [51]. They used the 42 years of simulation carried out with the hourly model Pasopl-20. They considered three Canadian cities, six load to collector ratio, two types of collector surface, four wall thicknesses, three wall thermal conductivities, and all other necessary parameters of a reference design. They obtained an annual correlation for each type of surface. These results were compared with the hourly and monthly models.

Sharma et al [52] proposed and examined a modified form of Trombe wall in which the glazing had been replaced by weather resistance material for winter heating and summer cooling in mixed climate conditions characterized by a mild winter and a relatively harsh dry summer. Their experimental results indicated the potential applications of such passive building architecture for mixed climate conditions.

Chapter 3

PHYSICAL MODEL AND ANALYSIS

3.1 Introduction

In the present chapter the physical model for a Trombe wall and the mathematical analysis used to solve the governing equations will be discussed.

First of all, the physical model of the problem for both steady and unsteady conditions are presented. The governing equations with their boundary conditions are simplified with certain assumptions. Using a transformations for these equations they are cast into a suitable form and the partial differential equations are transformed into ordinary differential equations. A new technique is used to solve these equations analytically and the results are to be compared with numerical and experimental published data.

3.2 Physical model and mathematical formulation

The physical model considered in this study is illustrated in Fig.(3-1).

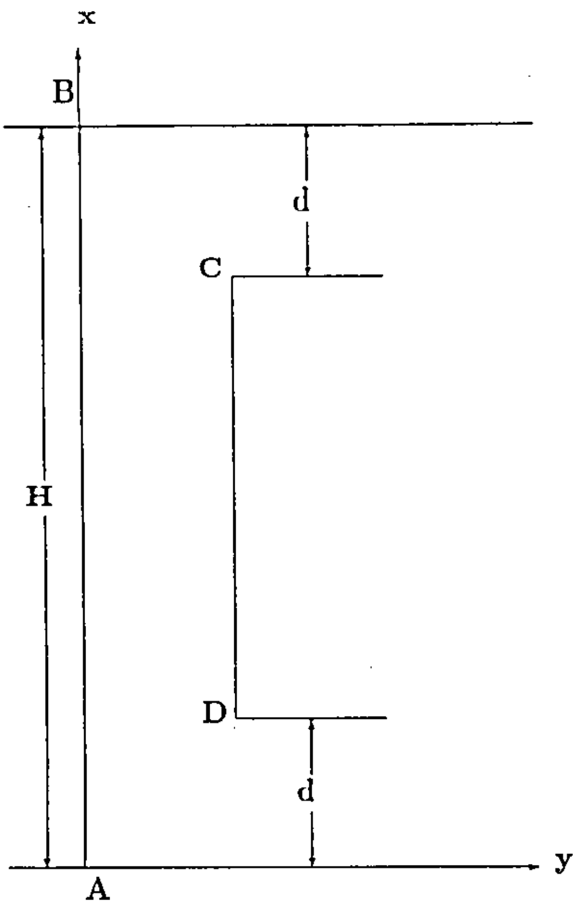

Fig.3-1 Schematic of the system

The exterior wall, AB, is maintained at a temperature T_w which may be constant or varying as the power law. The interior wall, CD, has a lower temperature. The fluid entering from the lower horizontal channel is heated by absorbing energy and circulating through the channel.

For the determination of the velocity profiles and temperature fields in a natural convection heat transfer process, the governing equations are:

- continuity

$$\frac{D\rho}{D\tau} + \rho \bigtriangledown . \vec{V} = 0 \tag{3.1}$$

- momentum

$$\rho \frac{D\vec{V}}{D\tau} = \vec{F} - \nabla P + \mu \nabla^2 \vec{V} + \mu/3 \nabla (\nabla . \vec{V}) \qquad (3.2)$$

- energy

$$\rho C_p \frac{DT}{D\tau} = \nabla . k \nabla T + \acute{Q} + \beta T \frac{DP}{D\tau} + \mu \Phi_v \qquad (3.3)$$

The basic force arises from the temperature difference in natural convection flows. The temperature variation results in a difference in density, which in turn results in a buoyancy force due to the presence of the body force field.

The temperature field is linked with the flow and the governing equations are coupled through the variation of the density ρ. The governing equations have to be solved simultaneously to give the distributions of the velocity and temperature fields, in space and time.

3.2.1 Approximations

The following approximations are made to simplify the governing equations.

1. Boussinesq approximation

   One could say that the governing equations for natural convection flow are of the types of coupled elliptic partial differential equations and are, therefore, of considerable complexity. The major problems in obtaining a solution to these equations lie in the inevitable variation of density ρ with temperature, or concentration, and in their partial, elliptic nature. The question of density variation is considered first and to do that we have to examine the magnitude and nature of the driving force. The density difference may be estimated as follows[53] :

$$\rho_a - \rho = \rho_a \beta (T - T_a) \tag{3.4}$$

or:

$$\rho = \rho_a (1 - \beta (T - T_a)) \tag{3.5}$$

With the Boussinesq approximations, the governing equations for constant properties become:

- continuity

$$\nabla . \vec{V} = 0 \tag{3.6}$$

- momentum

$$\rho \left(u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} \right) = -\vec{e} g \beta \rho (T - T_a) - \nabla P_d + \mu \nabla^2 \vec{V} \tag{3.7}$$

- energy

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \nabla^2 T + \acute{Q} + \beta T \frac{DP}{D\tau} + \mu \Phi_v \tag{3.8}$$

where,

T_a	-	ambient temperature
P_d	-	pressure due to the motion of the fluid
$\vec{e}$	-	unit vector in the direction of the gravitational field

For a vertical surface, with x along the surface and in the direction opposite to gravity, $\vec{e} = -\vec{i}$, where $\vec{i}$ is unit vector in the direction of x. The buoyancy term would appear only in the x component of the momentum equation. For an inclined surface, $\vec{e}$ would give rise to components in all directions.

2. Boundary-Layer Approximation.

The second approximation made in the governing equations (3.6 – 3.8) pertains to the extensively employed boundary-layer flow assumption. The fluid immediately adjacent to the surface is assumed to have no slip condition. The convective and diffusion components of energy and momentum transport are of the same order of magnitude in the boundary layer and, hence, the complete equations have to be considered. The pressure in the region beyond the boundary-layer is hydrostatic and the velocity outside the layer is zero. Using the order of magnitude analysis we can simplify these equations. The resulting boundary-layer equations governing the flow can be written for constant fluid properties as follows:

- continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.9}$$

- momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_a) + \frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) \tag{3.10}$$

- energy

$$\rho C_p(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = \acute{Q} + \beta T u\frac{\partial P_a}{\partial x} + \mu(\frac{\partial u}{\partial y})^2 + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) \tag{3.11}$$

Finally for steady, laminar boundary-layer neglecting the viscous dissipation and pressure term and with no heat generation the governing equations with their boundary conditions are:

- continuity.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.12}$$

- momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_a) + \nu\frac{\partial^2 u}{\partial y^2} \tag{3.13}$$

- energy

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3.14}$$

The boundary conditions are:

$$\left.\begin{array}{ll} y = 0: & u = 0, \quad v = 0, \quad T = T_w \\ y = y_\infty: & u = 0, \quad T = T_a \end{array}\right\} \tag{3.15}$$

For unsteady, laminar boundary layer the governing equations are given by:

- continuity.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.16}$$

- momentum

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_a) + \nu\frac{\partial^2 u}{\partial y^2} \tag{3.17}$$

- energy

$$\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3.18}$$

3.3 Similarity Variable Method

In this section the similarity transformations for the solution of convection problems are introduced.

The main problem in solving equations (3.12 – 3.14) lies in their partial differential nature. Both the velocity components (u) and (v) and the temperature (T) are functions of x and y. If, however, the dependence of x and y can be combined into a simple variable η the partial differential equations are transformed into a set of ordinary differential equations. Any reduction in the number of independent variables of a problem is an important simplification toward the solution of the problem, whether the problem is to be solved analytically or numerically.

The variable η is known to be the *similarity variable* of equations (3.12-3.14), the corresponding distributions being similar at all locations along the surface, when given in terms of η. In terms of η the reduction of equations (3.12-3.14) to ordinary differential equations is *a similarity transformation*, and the solution of these equations is *a similarity solution.* This solution has the property that two temperature profiles located at different x differ only by an x-dependent scale factor in y. That is, all temperature profiles become identical in a plot for θ/θ_0 versus η. It is from this fact that we get the name *similarity*.

The vertical wall under consideration is located at y=0, the coordinate y being indicative of the position in the boundary layer away from the wall surface. The wall is at temperature T_w, which may be a function of x, the vertical position along the surface, extending upwards from the leading edge, x=0. The temperature of air in the channel, T_a.

A stream function ψ is defined so that it satisfies the continuity equation.

$$u = \frac{\partial \psi}{\partial y} \quad and \quad v = -\frac{\partial \psi}{\partial x} \tag{3.19}$$

The variation of the wall temperature T_w may be expressed as :

$$T_w = T_a + A(x)$$

or:

$$T_w - T_a = A(x) \tag{3.20}$$

From the magnitude analysis of the governing equations, it was found that boundary layer thickness δ is proportional to $x/\sqrt[4]{Gr_x}$, where

$$Gr_x = \frac{g\beta x^3(T_w - T_a)}{\nu^2} \tag{3.21}$$

a similarity variable η may be expected from a combination of x and y, if η is taken proportional to y/δ. In natural convection, $\eta \sim (y\sqrt[4]{Gr_x})/x$ may be expected to give the desired similarity solution [55].

Now, the similarity variable η and a non-dimensional stream function f, which must be dependent only on η for success of the similarity method, may be expressed as:

$$\left.\begin{aligned} \eta(x,y) &= yC(x) \\ \psi &= \nu D(x)f(\eta) \end{aligned}\right\} \tag{3.22}$$

The temperature T is also generalized for the similarity solution, so that the generalized temperature θ is a function of η alone:

$$\theta = \frac{T - T_a}{T_w - T_a} \tag{3.23}$$

Our problem now is to determine the unknown functions $C(x)$ and $D(x)$, so that the temperature θ and the non-dimensional stream function f depend only on η. Using the above expressions $(3.19-3.23)$, we can find each term of the equations $(3.12-3.14)$:

$$
\left.
\begin{aligned}
u &= \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y} \\
&= C(x)\nu D(x)f' \\
-v &= \frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial x} \\
&= \nu D_x f' + yC_x\nu D(x)f' \\
\frac{\partial u}{\partial x} &= \frac{\partial u}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x} \\
&= \nu f'(C_xD + CD_x) + \nu yCC_xDf'' \\
\frac{\partial u}{\partial y} &= \frac{\partial u}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y} \\
&= \nu C^2Df'' \\
\frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial\eta}\left(\frac{\partial u}{\partial\eta}\right)\frac{\partial\eta}{\partial y} \\
&= \nu C^3Df''' \\
\frac{\partial T}{\partial x} &= A_x\theta + A\frac{\partial\theta}{\partial x} \\
&= A_x\theta + A\theta' yC_x \\
\frac{\partial T}{\partial y} &= \frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial y} \\
&= C\theta' \\
\frac{\partial^2 T}{\partial y^2} &= \frac{\partial}{\partial\eta}\left(\frac{\partial\theta}{\partial\eta}\right)\frac{\partial\eta}{\partial y} \\
&= C^2\theta''
\end{aligned}
\right\} \qquad (3.24)
$$

By substituting these equations into equations (3.12 – 3.14) we will get the following ordinary differential equations:

$$\underbrace{f'''}_{diffusion} + \underbrace{\frac{A}{DC^3}\frac{g\beta}{\nu^2}\theta}_{thermal-buoancy} + \underbrace{\frac{D_x}{C}ff'' - \left(\frac{D_x}{C} + \frac{DC_x}{C^2}\right)(f')^2}_{convection-terms} = 0 \qquad (3.25)$$

$$\underbrace{\theta''}_{diffusion} + \underbrace{Pr\frac{D_x}{C}f\theta' - \frac{DA_x}{CA}f'\theta}_{thermal-convection} = 0 \qquad (3.26)$$

Where the primes indicate the derivatives with respect to η and subscripts with respect to the subscripted variable. The possibility of a similarity solution now depends on the groups of A ,C, and D, obtained above, being independent of x and y. The boundary conditions must also depend only on η.

Similarity would demand that the parameters associated with the various terms be constant or functions of η alone. Since y does not appear in any of the groups, making it impossible to obtain η from them and these groups should be constants, the result is :

$$\left.\begin{aligned} \frac{A}{DC^3} &= const \\ \frac{D_x}{C} &= const \\ \frac{DC_x}{C^2} &= const \\ \frac{DA_x}{CA} &= const \end{aligned}\right\} \tag{3.27}$$

The required x dependence of A, C and D may be determined which would satisfy equation (3.27).

We will consider two cases corresponding to the wall temperature:

1. <u>A Constant Wall Temperature</u>

$$\left.\begin{aligned} T_w - T_a &= const \\ A(x) &= const \end{aligned}\right\} \tag{3.28}$$

Therefore, only three conditions have to be satisfied from equation (3.27) , which are:

$$\left.\begin{aligned} DC^3 &= const = M_1 \\ \frac{D_x}{C} &= const = M_2 \\ \frac{DC_x}{C^2} &= const = M_3 \end{aligned}\right\}$$

The terms of equations (3.12 – 3.14) can be estimated as follows:

$$\left.\begin{aligned}
u &= 4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f' x^{\frac{1}{2}} \\
-v &= 3\nu\sqrt[4]{\frac{g\beta(T_w-T_a)}{4\nu^2}^2} f x^{-\frac{1}{4}} - y x^{-\frac{1}{2}} f' \sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} \\
\frac{\partial u}{\partial x} &= 2x^{-\frac{1}{2}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f' - y x^{-\frac{3}{4}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^3} \\
\frac{\partial u}{\partial y} &= 4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^3\frac{1}{\nu^2}} f'' x^{\frac{1}{4}} \\
\frac{\partial^2 u}{\partial y^2} &= \frac{g\beta(T_w-T_a)}{\nu} f''' \\
\frac{\partial \theta}{\partial x} &= -\frac{1}{4} y x^{-\frac{5}{4}}\sqrt[4]{\frac{g\beta(T_w-T_a)}{4\nu^2}} \\
\frac{\partial \theta}{\partial y} &= x^{-\frac{1}{4}}\sqrt[4]{\frac{g\beta(T_w-T_a)}{4\nu^2}}\theta' \\
\frac{\partial^2 \theta}{\partial y^2} &= x^{-\frac{1}{2}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4\nu^2}\right)^2}\theta''
\end{aligned}\right\} \tag{3.32}$$

By substituting in equations (3.12 – 3.14) the above terms we get [54]:

$$f''' + 3ff'' - 2(f')^2 + \theta = 0 \tag{3.33}$$

$$\theta'' + 3Prf\theta' = 0 \tag{3.34}$$

The boundary conditions can be estimated using equation (3.32) by substituting for f' and f as follows:

$$f' = \frac{u}{4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} x^{\frac{1}{2}}}$$

$$f = \frac{-v + y x^{-\frac{1}{2}} f' \sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2}}{3\nu\sqrt[4]{\frac{g\beta(T_w-T_a)}{4\nu^2}} f x^{-\frac{1}{4}}}$$

From the above equations and equation (3.15) one could get that at $\eta = 0$ $u = 0$ then $f' = 0$ and when $v = 0$ the stream function $f = 0$.

2. The Temperature of wall is a function of x

Under this condition the temperature wall distribution is in the following form:

$$T_w - T_a = A(x)$$

The additional constraint of $\frac{DA_x}{AC} = const$ must be satisfied. We will consider the case when $A(x) = Nx^n$ i.e the wall temperature varies in terms of a power law form. As was shown in the isothermal wall condition, for the power law variation we can obtain that:

$$C(x) \sim x^{(n-1)/4} \quad and \quad D(x) \sim x^{(n+3)/4}$$

And in terms of Grashof number these can be expressed as:

$$\left.\begin{aligned} \eta &= y\sqrt[4]{\frac{g\beta}{4\nu^2}}\, x^{\frac{n-1}{4}} \\ \psi &= 4\nu\sqrt[4]{\frac{g\beta}{4\nu^2}}\, x^{\frac{n+3}{4}} f(\eta) \end{aligned}\right\} \tag{3.35}$$

and

$$C(x) = \frac{1}{x}\sqrt[4]{\frac{g\beta x^3(T_w - T_a)}{4\nu^2}} \tag{3.36}$$

$$D(x) = 4\sqrt[4]{\frac{g\beta x^3(T_w - T_a)}{4\nu^2}} \tag{3.37}$$

As defined above for velocity components u, and v will be as follows:

$$\left.\begin{aligned}
u &= 4\nu\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^2}\,x^{\frac{2n+2}{4}}f' \\
-v &= (n+3)x^{\frac{n-1}{4}}\nu\sqrt[4]{\frac{g\beta}{4\nu^2}}\,f \\
&+(n-1)x^{\frac{2n-2}{4}}\nu y\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^2}\,f' \\
\frac{\partial u}{\partial x} &= (2n+2)\nu x^{\frac{2n-2}{4}}\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^2}\,f' \\
&+(n-1)\nu y x^{\frac{3n-3}{4}}\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^3}\,f'' \\
\frac{\partial u}{\partial y} &= 4\nu\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^3}\,x^{\frac{3n+1}{4}}f'' \\
\frac{\partial^2 u}{\partial y^2} &= g\beta\frac{x^n}{\nu}f'''
\end{aligned}\right\} \qquad (3.38)$$

By substituting in equations $(3.12-3.13)$ the above terms of equation (3.38) we get for the power law distribution :

$$f''' + (n+3)ff'' - 2(n+1)(f')^2 + \theta = 0 \qquad (3.39)$$

The energy equation (3.14) may be transformed as follows:

$$\left.\begin{aligned}
&u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}\\[2ex]
&u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}+u\theta\frac{1}{\Delta T}\frac{d\Delta T}{dx}=\alpha\frac{\partial^2 \theta}{\partial y^2}\\[2ex]
&4\nu\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^2}x^{\frac{2n+2}{4}}f'\frac{n-1}{4}x^{\frac{n-5}{4}}y\sqrt[4]{\frac{g\beta}{4\nu^2}}\theta'\\
&+4\nu\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^2}x^{\frac{2n+2}{4}}f'\theta\frac{1}{x^n}nx^{(n-1)}\\
&-(n+3)x^{\frac{n-1}{4}}\nu\sqrt[4]{\frac{g\beta}{4\nu^2}}f\sqrt[4]{\frac{g\beta}{4\nu^2}}\theta' x^{\frac{n-1}{4}}\\
&-(n-1)x^{(2n-2)/4}\nu y\sqrt[4]{\left(\frac{g\beta}{4\nu^2}\right)^2}f'\sqrt[4]{\frac{g\beta}{4\nu^2}}\theta' x^{\frac{n-1}{4}}\\
&=\frac{\alpha}{\nu}\sqrt[4]{\frac{g\beta}{4\nu^2}}x^{\frac{n-1}{2}}\theta''\\[2ex]
&(n-1)\sqrt[4]{\left(\frac{g\beta}{4}\right)^3\frac{1}{\nu^2}}x^{\frac{3n-3}{4}}f'y\theta'\\
&-(n+3)\sqrt[4]{\left(\frac{g\beta}{4}\right)^2}x^{\frac{n-1}{2}}f\theta'\\
&-(n-1)\sqrt[4]{\left(\frac{g\beta}{4}\right)^3\frac{1}{\nu^2}}x^{\frac{3n-3}{4}}f'y\theta'\\
&+4n\sqrt[4]{\left(\frac{g\beta}{4}\right)^2}x^{\frac{n-1}{2}}f'\theta\\
&=\frac{\alpha}{\nu}\sqrt[4]{\left(\frac{g\beta}{4}\right)^2}x^{\frac{n-1}{2}}\theta''
\end{aligned}\right\}\qquad (3.40)$$

$$\theta''+(n+3)Prf\theta'-4nPrf'\theta=0 \qquad (3.41)$$

3. <u>Unsteady Laminar Case</u>

The governing equations for unsteady laminar boundary layer (3.16–3.18) are first cast to a non-dimensionless form using the following dimensionless quantities which are defined according to the procedure indicated in [35]:

$$\left.\begin{array}{ll} X = \frac{x}{L}; & Y = \frac{y}{L}; \\ t = \frac{\alpha\tau}{L^2}; & V = \frac{vL}{\alpha}; \\ U = \frac{ul}{\alpha}; & G = \frac{g\beta L^3 (T - T_a)}{\nu\alpha} \end{array}\right\} \quad (3.42)$$

where,

ψ	-	stream function
L	-	characteristic length
G	-	generalized Rayliegh number
ν	-	kinematic viscosity
α	-	thermal diffusivity
β	-	thermal coefficient of expansion

The resulting non-dimensionless equations are:

- **continuity.**

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (3.43)$$

- **momentum**

$$\left.\begin{array}{rcl} \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} & = & \frac{g\beta(T - T_a)L^3}{\alpha^2} + Pr\frac{\partial^2 U}{\partial Y^2} \\ & = & PrG + Pr\frac{\partial^2 U}{\partial Y^2} \end{array}\right\} \quad (3.44)$$

- **energy**

$$G_t + UG_X + VG_Y = G_{YY} \quad (3.45)$$

With the boundary conditions:

$$\left.\begin{aligned} Y &= 0: \\ \psi &= 0 \\ \psi_y &= 0 \\ G &= G_w(X,t) \\ Y &= Y\infty: \\ \psi_y &= 0 \\ G &= 0 \end{aligned}\right\} \quad (3.46)$$

First of all we introduce a similarity variable η in the form:

$$\eta = Y\phi_1(X,t) \quad (3.47)$$

and the stream function ψ and the temperature G are [35]

$$\psi = \phi_2(X,t)f(\eta) \quad (3.48)$$

$$\theta(\eta) = \frac{G}{G_w} \quad (3.49)$$

Equations $(3.43 - 3.45)$ are reduced to :

- **momentum**

$$Prf''' - d_1\eta f'' - (d_1 + d_2)f' + d_4 ff'' - (d_3 + d_4)(f')^2 + Prd_5\theta = 0 \quad (3.50)$$

- **energy**

$$\theta'' - (d_1\eta - d_4 f)\theta' - (d_6 + d_7 f')\theta = 0 \quad (3.51)$$

where,

$$d_1 = \frac{(\phi_1)_t}{\phi_1{}^3} \quad (3.52)$$

$$d_2 = \frac{(\phi_2)_t}{\phi_1{}^2\phi_2} \tag{3.53}$$

$$d_3 = \frac{\phi_2(\phi_1)_X}{\phi_1{}^2} \tag{3.54}$$

$$d_4 = \frac{(\phi_2)_X}{\phi_1} \tag{3.55}$$

$$d_5 = \frac{G_w}{\phi_1{}^3\phi_2} \tag{3.56}$$

$$d_6 = \frac{(G_w)_t}{G_w\phi_1{}^2} \tag{3.57}$$

$$d_7 = \frac{(G_w)_X\phi_2}{G_w\phi_1} \tag{3.58}$$

The boundary conditions are:

$$\left.\begin{array}{lcl} \eta & = & 0: \\ f(0) & = & 0 \\ f'(0) & = & 0 \\ \theta(0) & = & 1 \\ \eta & = & \eta_\infty : \\ f'(\eta_\infty) & = & 0 \\ \theta(\eta_\infty) & = & 0 \end{array}\right\} \tag{3.59}$$

The similarity formulations are possible if and only if d_1 through d_7 are all constants.

ϕ_1 and ϕ_2 had to be solved from the differential equations (3.52 – 3.55) . Then equation (3.56) yields the surface temperature variation G_w for which a similarity solution is possible. Integrating equation (3.52) yields:

$$\phi_1 = (A(X) - 2d_1t)^{-\frac{1}{2}} \tag{3.60}$$

And from (3.52) and (3.53) one may get:

$$\phi_2 = B(X)((A(X) - 2d_1t)^{-\frac{\delta}{2}} \tag{3.61}$$

where,

$$\delta = \tfrac{d_2}{d_1}, \qquad d_1 \neq 0$$

But from equation (3.54) and (3.55) we get:

$$\phi_2 = C(t)\phi_1^{\epsilon} \tag{3.62}$$

where,

$$\epsilon = \tfrac{d_4}{d_3}, \qquad d_3 \neq 0$$

Solving equation (3.54) using equation (3.62) one could obtain:

$$\phi_1 = [\frac{d_3(\epsilon - 1)}{C(t)}X + D(t)]^{\frac{1}{\epsilon-1}} \tag{3.63}$$

And from equation (3.62) we get:

$$\phi_2 = C(t)[\frac{d_3(\epsilon - 1)}{C(t)}X + D(t)]^{\frac{\epsilon}{\epsilon-1}} \tag{3.64}$$

Equations $(3.61 - 3.64)$ can be solved if and only if :

$$\left.\begin{aligned} &\epsilon = \delta = -1 \\ &B(X) = C(t) = d_8 \\ &A(X) = -(\tfrac{2d_3}{d_8})X \\ &D(t) = -2d_1t \end{aligned}\right\} \tag{3.65}$$

Finally ϕ_1 and ϕ_2 can be obtained as follows:

$$\phi_1 = (-\frac{2d_3}{d_8})X - 2d_1t)^{-\frac{1}{2}} \tag{3.66}$$

$$\phi_2 = d_8(-\frac{2d_3}{d_8}X - 2d_1 t)^{\frac{1}{2}} \tag{3.67}$$

and from equation (3.56) the wall temperature distribution is obtained:

$$G_w = d_5 d_8[-\frac{2d_3}{d_8}X - 2d_1 t]^{-1} \tag{3.68}$$

The constant which we have to keep as a parameter in the reduced similarity equation is d_1, where d_5 and d_3 may be assumed arbitrary. Thus we have

$$\left.\begin{array}{rcl} d_1 & = & -d_2 \\ d_3 & = & -d_4 \end{array}\right\}$$

The following equations are to be verified [35] :

$$d_6 = 2d_2 + 3d_1 \tag{3.69}$$

$$d_7 = 2d_4 + 3d_3 \tag{3.70}$$

From equations (3.69) and (3.70) we get:

$$\left.\begin{array}{rcl} d_6 & = & d_1 \\ d_7 & = & d_3 \end{array}\right\}$$

Taking $d_3 = -1$ and $d_5 = 1$ the similarity equations for the flow may now be written as follows:

$$Prf''' - d_1\eta f'' + ff'' + Pr\theta = 0 \tag{3.71}$$

$$\theta'' - (d_1\eta - f)\theta' - (d_1 - f')\theta = 0 \tag{3.72}$$

The similarity formulation is not valid if the quantity $(\frac{2X}{d_8} - 2d_1 t)$ is non-positive. We have to be careful to choose d_1 such that this term will not be a negative.

3.4 Method Of Solution

3.4.1 Constant Wall Temperature

The sets of equations that were derived in the foregoing paragraphs are nonlinear ordinary differential equations to find their solution we assume the solution for the stream function (f) as a function of η which satisfy the boundary condition, that is:

$$\text{at } \eta = 0 \quad f = 0$$

The simplest form of this guessed function is

$$f(\eta) = \alpha\eta \tag{3.73}$$

In the equations (3.33–3.34) we put instead of "$(f)''$" in the second term the guessed function and one of the derivatives "$(f')^2$" is replaced by the first derivative of the guessed function. Thus, the equations (3.33 – 3.34) reduce to:

$$f''' + 3\alpha\eta f'' - 2\alpha f' + \theta = 0 \tag{3.74}$$

and

$$\theta'' + 3\alpha Pr\eta\theta' = 0 \tag{3.75}$$

This for the isothermal wall conditions, i.e $T_w = const$. Due to the inevitable coupling between the above equations we will solve equation (3.75) first to get the temperature distribution, then it is substituted in equation (3.74) to get the velocity profile, which relates with the stream function.

The solution of equation (3.75) can be obtained in terms of Kummer's function. It is more convenient to find the solution in terms of this function because it is easy to get the particular solution for the nonhomogenous part of the momentum equation. Also to identify the solutions for all equations. Kummer function is the solution of the confluent hypergeometric equation [56].

$$1F1[a;b;x] = \sum_{i=1}^{n} \frac{(a)_n x^n}{(b)_n n!} \tag{3.76}$$

where,

$$\left.\begin{aligned} (a)_n &= a(a+1)(a+2)\cdots(a+n-1) \\ (b)_n &= b(b+1)(b+2)\cdots(b+n-1) \\ n! &= n(n-1)(n-2)\cdots 1 \end{aligned}\right\} \tag{3.77}$$

This series is absolutely convergent for all values of a, b, and x, real or complex.

Wittaker was the first to study the following equation [57]:

$$\frac{d^2y}{dx^2} + (\frac{1}{4} + \frac{k}{x} + \frac{\frac{1}{4} - m^2}{x^2})y = 0 \tag{3.78}$$

One of this equation's solutions is Wittaker's function:

$$y = M_{k,m}(x) \tag{3.79}$$

Wittaker's function is related to Kummer's function as follows:

$$M_{k,m}(x) = x^{\frac{1}{2}+m} e^{-\frac{1}{2}x} 1F1[\frac{1}{2} + m - k; 1 + 2m; x] \tag{3.80}$$

Equation (3.75) is of the form :

$$y'' + a_0 x y' + (a_1 x^2 + b_1)y = 0 \tag{3.81}$$

with the solution in the form:

$$y = \frac{2e^{-\frac{a_0}{4}x^2}}{\sqrt{\pi x}} M_{k,\pm\frac{1}{4}}(x^2\sqrt{(\frac{a_0^2}{4} - a_1)}) \quad (3.82)$$

where,

$$k = \frac{\frac{1}{2}(b_1 - \frac{1}{2}a_0)}{\sqrt{(a_0^2 - 4a_1)}} \quad (3.83)$$

By comparison the coefficients of equation (3.75) with equation (3.81) we get the following:

$$\left.\begin{aligned} a_0 &= 3\alpha Pr \\ a_1 &= 0 \\ b_1 &= 0 \\ k &= -\tfrac{1}{4} \end{aligned}\right\} \quad (3.84)$$

and the solution for θ is:

$$\theta = \frac{2}{\sqrt{\pi\eta}} e^{-\frac{3\alpha Pr}{4}\eta^2}\left[C_1' M_{-\frac{1}{4},\frac{1}{4}}(\frac{3\alpha Pr}{2}\eta^2) + C_2' M_{-\frac{1}{4},-\frac{1}{4}}(\frac{3\alpha Pr}{2}\eta^2)\right] \quad (3.85)$$

Using the relations between confluent hypergeometric functions, Equation (3.80) and the first theorem of Kummer of their own properties [56], which states that:

$$e^{-x} 1F1[a; b; x] = 1F1[b - a; b; -x] \quad (3.86)$$

one could get:

$$\theta = C_1 \eta 1F1[\frac{1}{2}; \frac{3}{2}; -\frac{3\alpha Pr}{2}\eta^2] + C_2 \quad (3.87)$$

C_1 and C_2 are arbitrary constants which can be determined using the boundary conditions:

$$\theta(0) = 1 \qquad \theta(\eta_\infty) = 0$$

Thus the constants are:

$$\left.\begin{aligned} C_1 &= \frac{-1}{\eta_\infty 1F1[\frac{1}{2};\frac{3}{2};-\frac{3\alpha Pr}{2}\eta_\infty^2]} \\ C_2 &= 1 \end{aligned}\right\} \tag{3.88}$$

The final solution for the temperature distribution θ is:

$$\theta = 1 - \frac{\eta}{\eta_\infty 1F1[\frac{1}{2};\frac{3}{2};-\frac{3\alpha Pr}{2}\eta_\infty^2]} 1F1[\frac{1}{2};\frac{3}{2};-\frac{3\alpha Pr}{2}\eta^2] \tag{3.89}$$

Since the temperature distribution is known as a function of η, Equation (3.74) will be a non-homogenous ordinary differential equation, whose solution consists of two parts: the complementary solution and the particular one for the non-homogenous part.

Equation (3.74) is first reduced to a second order differential equation by using the reduction of order method as follows:

$$P = \frac{df}{d\eta}; \quad \frac{dP}{d\eta} = \frac{d^2 f}{d\eta^2}; \quad \frac{d^2 P}{d\eta^2} = \frac{d^3 f}{d\eta^3} \tag{3.90}$$

Equation (3.74) reduces to:

$$P'' + 3\alpha\eta P' - 2\alpha P = 0 \tag{3.91}$$

The complementary solution for this equation P_c is obtained in the same way as for the temperature solution. By comparison the coefficients with these in equation (3.81) we get:

$$\left.\begin{aligned} a_0 &= 3\alpha \\ b_1 &= -2\alpha \\ k &= -\tfrac{7}{12} \end{aligned}\right\} \tag{3.92}$$

and the solution for P_c is then:

$$P_c = \frac{2}{\sqrt{\pi\eta}} e^{-\frac{3\alpha}{4}\eta^2} \left[C_3' M_{-\frac{7}{4},\frac{1}{4}}(\frac{3\alpha}{2}\eta^2) + C_4' M_{-\frac{7}{4},-\frac{1}{4}}(\frac{3\alpha}{2}\eta^2)\right] \tag{3.93}$$

which yields:

$$P_c = C_3''\eta e^{-\frac{3\alpha}{2}\eta^2}1F1[\frac{4}{3};\frac{3}{2};\frac{3\alpha}{2}\eta^2] + C_4''e^{-\frac{3\alpha}{2}\eta^2}1F1[\frac{5}{6};\frac{1}{2};\frac{3\alpha}{2}\eta^2] \qquad (3.94)$$

Integrating equation (3.94) with respect to η we get the stream function distribution as follows:

$$f_c = C_3'''e^{-\frac{3\alpha}{2}\eta^2}1F1[\frac{4}{3};\frac{1}{2};\frac{3\alpha}{2}\eta^2] + C_4'''\eta e^{-\frac{3\alpha}{2}\eta^2}1F1[\frac{5}{6};\frac{1}{2};\frac{3\alpha}{2}\eta^2] + C_5 \qquad (3.95)$$

Equation (3.95) reduces to :

$$f_c = C_3 1F1[-\frac{5}{6};\frac{1}{2};-\frac{3\alpha}{2}\eta^2] + C_4\eta 1F1[-\frac{1}{3};\frac{3}{2};-\frac{3\alpha}{2}\eta^2] + C_5 \qquad (3.96)$$

The derivative of equation (3.96) may be written as:

$$f_c' = 5\alpha\eta C_3 1F1[\frac{1}{6};\frac{3}{2};-\frac{3\alpha}{2}\eta^2] + C_4 1F1[-\frac{1}{3};\frac{1}{2};-\frac{3\alpha}{2}\eta^2] \qquad (3.97)$$

The complete solution for the equation (3.74) is the sum of the complementary solution and the particular one for the non-homogenous part (f_p). To find this solution we assume it in the form of a power series solution:

$$f_p = \sum_{i=0}^{\infty} A_m\eta^{m+1} \qquad (3.98)$$

and then we get the first, second and third derivatives as follows:

$$\left.\begin{aligned} f_p' &= \textstyle\sum_{m=0}^{\infty}(m+1)A_m\eta^m \\ f_p'' &= \textstyle\sum_{m=1}^{\infty} m(m+1)A_m\eta^{m-1} \\ f_p''' &= \textstyle\sum_{m=2}^{\infty} m(m+1)(m-1)A_m\eta^{m-2} \end{aligned}\right\}$$

By substituting these terms in equation (3.74) we get:

$$\left.\begin{aligned}&6A_2 - 2\alpha A_0\\&+\sum_{m=1}^{\infty}[(m+1)(m+2)(m+3)A_{(m+2)}+\\&(3\alpha m(m+1) - 2\alpha(m+1))A_m]\eta^m\\&= C_1\sum_{n=1}^{\infty}(-1)^n\frac{(\frac{1}{2})_n}{(\frac{3}{2})_n n!}(\sqrt{\frac{3\alpha Pr}{2}})^n\eta^{(n+1)} - 1\end{aligned}\right\} \quad (3.99)$$

By comparing the coefficient of both sides of equation (3.99) one could obtain the coefficient of the particular part of the solution namely A_m.

For $m = odd$:

$$A_m = \frac{1}{(m+1)(3m-2)\alpha}\left[(-1)^k(\tfrac{1}{2})_k\frac{C_1}{k!(\frac{3}{2})_k}(\tfrac{3\alpha Pr}{2})^k - \beta A_{(m+2)}\right] \quad (3.100)$$

for $m = even$:

$$A_m = 0 \quad (3.101)$$

where,

$$k = \tfrac{m-1}{2}; \qquad \beta = (m+1)(m+2)(m+3)$$

Finally the complete solution for the stream function and the velocity distributions are:

$$f = C_3 1F1[-\frac{5}{6};\frac{1}{2};-\frac{3\alpha}{2}\eta^2] + C_4\eta 1F1[-\frac{1}{3};\frac{3}{2};-\frac{3\alpha}{2}\eta^2] + f_p + C_5 \quad (3.102)$$

$$f' = 5\alpha\eta C_3 1F1[\frac{1}{6};\frac{3}{2};-\frac{3\alpha}{2}\eta^2] + C_4 1F1[-\frac{1}{3};\frac{1}{2};-\frac{3\alpha}{2}\eta^2] + f_p' \quad (3.103)$$

The boundary conditions related to these equations are:

$$\left.\begin{aligned} \eta &= 0 : \\ f &= 0 \\ f' &= 0 \\ \eta &= \eta_\infty : \\ f' &= 0 \end{aligned}\right\} \qquad (3.104)$$

The constants C_3, C_4 and C_5 will be determined using these boundary conditions.

$$\left.\begin{aligned} & f(0) = C_3 + C_5 = 0 \\ & f'(0) = C_4 + f_p' = 0 \\ & f'(\eta_\infty) = 5\alpha\eta_\infty C_3 1F1[\tfrac{1}{6}; \tfrac{3}{2}; -\tfrac{3\alpha}{2}\eta_\infty^2] \\ & +C_4 1F1[-\tfrac{1}{3}; \tfrac{1}{2}; -\tfrac{3\alpha}{2}\eta_\infty^2] + f_p'(\eta_\infty) = 0 \end{aligned}\right\} \qquad (3.105)$$

Solving the above equation (3.105) one could get:

$$\left.\begin{aligned} C_3 &= -C_5 \\ C_4 &= -f_p'(0) \\ C_3 &= -\frac{C_4 1F1[\frac{1}{3}; \frac{1}{2}; -\frac{3\alpha}{2}\eta^2] + f'(\eta_\infty)}{5\alpha\eta_\infty 1F1[\frac{1}{6}; \frac{3}{2}; -\frac{3\alpha}{2}\eta_\infty^2]} \end{aligned}\right\} \qquad (3.106)$$

3.4.2 The Power Law Distribution For The wall Temperature

By introducing the guessed function for the stream function equation (3.73) for (f) and (f') into equations (3.39) and (3.41) they yield:

$$f''' + (n+3)\alpha\eta f'' - 2(n+1)f' + \theta = 0 \tag{3.107}$$

$$\theta'' + (n+3)\alpha\eta Pr\theta' - 4nPr\alpha\theta = 0 \tag{3.108}$$

The solution for equation (3.108) is obtained by comparison of the coefficient of this equation with equation (3.81); therefore:

$$\left.\begin{aligned} a_0 &= (n+3)\alpha Pr \\ a_1 &= 0 \\ b_1 &= -4nPr\alpha \\ k &= -\frac{9n+3}{4(n+3)} \end{aligned}\right\} \tag{3.109}$$

The solution for θ in the form of Wittaker function is as follows:

$$\left.\begin{aligned} \theta &= e^{-\frac{n+3}{4}\alpha Pr\eta^2}\frac{1}{\sqrt{\eta}}[C_6' M_{-\frac{9n+3}{4(n+3)},-\frac{1}{4}}(\tfrac{n+3}{2}\alpha Pr\eta^2) \\ &+C_7' M_{-\frac{9n+3}{4(n+3)},\frac{1}{4}}(\tfrac{n+3}{2}\alpha Pr\eta^2)] \end{aligned}\right\} \tag{3.110}$$

This is reduced by using the relations of the hypergeometric functions to the following form:

$$\left.\begin{aligned} \theta &= C_6 1F1[-\tfrac{2n}{n+3};\tfrac{1}{2};-\tfrac{n+3}{2}\alpha Pr\eta^2] \\ &+C_7\eta 1F1[\tfrac{3(1-n)}{2(n+3)};\tfrac{3}{2};-\tfrac{n+3}{2}\alpha Pr\eta^2] \end{aligned}\right\} \tag{3.111}$$

The boundary conditions related to this equation are:

$$\left.\begin{aligned} \theta(0) &= 1 \\ \theta(\eta_\infty) &= 0 \end{aligned}\right\} \tag{3.112}$$

Thus:

$$\left.\begin{aligned} 1 = & \; C_6 \\ 0 = & \; 1F1[-\tfrac{2n}{n+3};\tfrac{1}{2};-\tfrac{n+3}{2}\alpha Pr\eta_{\infty}^2] \\ & +C_7\eta_{\infty}1F1[\tfrac{3(1-n)}{2(n+3)};\tfrac{3}{2};-\tfrac{n+3}{2}\alpha Pr\eta_{\infty}^2] \end{aligned}\right\} \quad (3.113)$$

Solving the system of equations (3.113) one could get:

$$\left.\begin{aligned} C_6 &= 1 \\ C_7 &= -\frac{1F1[-\frac{2n}{n+3};\frac{1}{2};-\frac{n+3}{2}\alpha Pr\eta_{\infty}^2]}{\eta_{\infty}1F1[\frac{3(1-n)}{2(n+3)};\frac{3}{2};-\frac{n+3}{2}\alpha Pr\eta_{\infty}^2]} \end{aligned}\right\} \quad (3.114)$$

The temperature distribution is obtained as follows:

$$\left.\begin{aligned} \theta = & \; 1F1[-\tfrac{2n}{n+3};\tfrac{1}{2};-\tfrac{n+3}{2}\alpha Pr\eta^2] \\ & -\frac{1F1[-\frac{2n}{n+3};\frac{1}{2};-\frac{n+3}{2}\alpha Pr\eta_{\infty}^2]}{\eta_{\infty}1F1[\frac{3(1-n)}{2(n+3)};\frac{3}{2};-\frac{n+3}{2}\alpha Pr\eta_{\infty}^2]}\eta 1F1[\tfrac{3(1-n)}{2(n+3)};\tfrac{3}{2};-\tfrac{n+3}{2}\alpha Pr\eta^2] \end{aligned}\right\} \quad (3.115)$$

As we had defined θ equation (3.111) is now a non-homogenous ordinary differential equation, whose solution consists of two components: the complementary solution for the homogenous part and the particular solution for the non-homogenous one.

To find the complementary part of the solution the homogenous part of equation (3.111) is first reduced to a second order ordinary differential equation by using the reduction of order method as in equation (3.90) then we get:

$$P'' + (n+3)\alpha\eta P' - 2(n+1)\alpha P = 0 \quad (3.116)$$

By comparison the coefficient of this equation with equation (3.81) we get:

$$\left.\begin{array}{lcl} a_0 & = & (n+3)\alpha \\ a_1 & = & 0 \\ b_1 & = & -2(n+1)\alpha \\ k & = & -\frac{5n+7}{4(n+3)} \end{array}\right\} \quad (3.117)$$

The solution of equation (3.116) is

$$\left.\begin{array}{rl} P = & \frac{df}{d\eta} \\ = & C_8' e^{-\frac{n+3}{2}\alpha\eta^2} 1F1[\frac{3n+5}{2(n+3)}; \frac{1}{2}; \frac{n+3}{2}\alpha\eta^2] \\ & +C_9' \eta e^{-\frac{n+3}{2}\alpha\eta^2} 1F1[\frac{2(n+2)}{n+3}; \frac{3}{2}; \frac{n+3}{2}\alpha\eta^2] \end{array}\right\} \quad (3.118)$$

Integrating this equation we get the complementary solution of the stream function namely f_c:

$$\left.\begin{array}{rl} f_c = & C_8 \eta 1F1[-\frac{n+1}{n+3}; \frac{3}{2}; -\frac{n+3}{2}\alpha\eta^2] \\ & +C_9 1F1[-\frac{3n+5}{2(n+3)}; \frac{1}{2}; -\frac{n+3}{2}\alpha\eta^2] + C_{10} \end{array}\right\} \quad (3.119)$$

The first derivative of this function can be obtained using the properties of the hypergeometric functions. Thus:

$$\left.\begin{array}{rl} f_c' = & C_8 1F1[-\frac{n+1}{n+3}; \frac{1}{2}; -\frac{n+3}{2}\alpha\eta^2] \\ & +(3n+5)\alpha\eta C_9 1F1[-\frac{1-n}{2(n+3)}; \frac{3}{2}; -\frac{n+3}{2}\alpha\eta^2] \end{array}\right\} \quad (3.120)$$

The particular part of solution is to be found in the same way as for the case of constant wall temperature distribution as described in section (3.3). The coefficient A_m are to be defined as follows:

for m = even

$$\left.\begin{array}{rl} A_m = & \frac{1}{(m+1)\alpha((m-2)n+3m-2)} \\ & [(-1)^i(-\frac{2n}{n+3})_i \frac{1}{i!}(\frac{1}{2})_i(\frac{n+3}{2}Pr\alpha)^i - \beta A_{m+2}] \end{array}\right\} \quad (3.121)$$

where,

$$i = \frac{m}{2} \qquad and \qquad \beta = (m+1)(m+2)(m+3)$$

for m=odd

$$\left.\begin{aligned} A_m = & \frac{1}{(m+1)\alpha((m-2)n+3m-2)} \\ & [(-1)^i(\frac{3(1-n)}{2(n+3)})_i \frac{C_7}{i!(\frac{1}{2})_i}(\frac{n+3}{2}\alpha Pr)^i - \beta A_{m+2}] \end{aligned}\right\} \quad (3.122)$$

where,

$$i = \frac{m-1}{2} \qquad and \qquad \beta = (m+1)(m+2)(m+3)$$

The complete solution for the stream function in the case of the power law temperature distribution is now:

$$\left.\begin{aligned} f = & C_8\eta 1F1[-\frac{n+1}{n+3}; \frac{3}{2}; -\frac{n+3}{2}\alpha\eta^2] \\ & +C_9 1F1[-\frac{3n+5}{2(n+3)}; \frac{1}{2}; -\frac{n+3}{2}\alpha\eta^2] + C_{10} + f_p \end{aligned}\right\} \quad (3.123)$$

and the derivative of this function is:

$$\left.\begin{aligned} f' = & C_8 1F1[-\frac{n+1}{n+3}; \frac{1}{2}; -\frac{n+3}{2}\alpha\eta^2] \\ & +(3n+5)\alpha\eta C_9 1F1[-\frac{1-n}{2(n+3)}; \frac{3}{2}; -\frac{n+3}{2}\alpha\eta^2] + f_p' \end{aligned}\right\} \quad (3.124)$$

To find out the constants C_8, C_9 and C_{10} we use the boundary conditions corresponding to this case which are:

$$\left.\begin{aligned} & f(0) = C_9 + C_{10} = 0 \\ & f'(0) = C_8 + f_p' = 0 \\ & f'(\eta_\infty) = (3n+5)\alpha\eta_\infty C_9 1F1[\frac{1-n}{2(n+3)}; \frac{3}{2}; -\frac{n+3}{2}\alpha\eta_\infty^2] \\ & +C_8 1F1[-\frac{n+1}{n+3}; \frac{1}{2}; -\frac{n+3}{2}\alpha\eta_\infty^2] + f_p'(\eta_\infty) = 0 \end{aligned}\right\} \quad (3.125)$$

Solving equation (3.125) one could get:

$$\left.\begin{aligned} C_8 = & -A_0 \\ C_9 = & -\frac{-A_0 1F1[-\frac{n+1}{n+3}; \frac{1}{2}; -\frac{n+3}{2}\alpha\eta_\infty^2] + f_p'(\eta_\infty)}{(3n+5)\alpha\eta_\infty 1F1[\frac{1-n}{2(n+3)}; \frac{3}{2}; -\frac{n+3}{2}\alpha\eta_\infty^2]} \\ C_{10} = & \frac{-A_0 1F1[-\frac{n+1}{n+3}; \frac{1}{2}; -\frac{n+3}{2}\alpha\eta_\infty^2] + f_p'(\eta_\infty)}{(3n+5)\alpha\eta_\infty 1F1[\frac{1-n}{2(n+3)}; \frac{3}{2}; -\frac{n+3}{2}\alpha\eta_\infty^2]} \end{aligned}\right\} \quad (3.126)$$

3.4.3 Unsteady Laminar Case

Equations (3.71) and (3.72) are to be solved in the same way as the above two cases. The resulting equations after substituting the guessed function equation (3.73) and its derivative in equations (3.71) and (3.72) are:

$$f''' - \frac{1}{Pr}(d_1 - \alpha)\eta f'' + \theta = 0 \qquad (3.127)$$

$$\theta'' - (d_1 - \alpha)\eta\theta - (d_1 - \alpha)\theta = 0 \qquad (3.128)$$

By comparing equation (3.128) with equation (3.81) we get:

$$\left.\begin{array}{lcl} a_0 & = & -(d_1 - \alpha) \\ a_1 & = & 0 \\ b_1 & = & -(d_1 - \alpha) \\ k = \frac{1}{4} & & \end{array}\right\} \qquad (3.129)$$

and the solution for θ is:

$$\left.\begin{array}{l} \theta = \frac{2}{\sqrt{\pi\eta}} e^{\frac{(d_1-\alpha)}{4}\eta^2} [C'_{11} M_{\frac{1}{4},\frac{1}{4}}(\frac{-(d_1-\alpha)}{2}\eta^2) \\ +C'_{12} M_{\frac{1}{4},-\frac{1}{4}}(\frac{-(d_1-\alpha)}{2}\eta^2)] \end{array}\right\} \qquad (3.130)$$

This is reduced to:

$$\theta = C_{11}\eta 1F1[1; \frac{3}{2}; \frac{(d_1 - \alpha)}{2}\eta^2] + C_{12} 1F1[\frac{1}{2}; \frac{1}{2}; \frac{(d_1 - \alpha)}{2}\eta^2] \qquad (3.131)$$

Using the boundary conditions specified by equation (3.59) one could find the arbitrary constants C_{11} and C_{12}, thus:

$$\left.\begin{array}{l} C_{12} = 1 \\ C_{11} = -\frac{1F1[\frac{1}{2};\frac{1}{2};\frac{(d_1-\alpha)}{2}\eta_\infty^2]}{\eta_\infty 1F1[1;\frac{3}{2};\frac{(d_1-\alpha)}{2}\eta_\infty^2]} \end{array}\right\} \qquad (3.132)$$

The solution for the stream function is obtained by solving equation (3.127). As was mentioned before first this equation is first reduced to a second order ordinary differential equation using equation (3.90). The complementary solution is found by comparing the resulting equation with equation (3.81), hence one may get:

$$\left.\begin{aligned} a_0 &= -\tfrac{d_1-\alpha}{Pr} \\ a_1 &= 0 \\ b_1 &= 0 \\ k &= -\tfrac{1}{4} \end{aligned}\right\} \qquad (3.133)$$

Thus,

$$P = \frac{2}{\sqrt{\eta}} e^{\frac{d_1-\alpha}{Pr}\eta^2} \left[C_{13}' M_{-\frac{1}{4},-\frac{1}{4}}(-\frac{d_1-\alpha}{2Pr}\eta^2) + C_{14}' M_{-\frac{1}{4},\frac{1}{4}}(-\frac{d_1-\alpha}{2Pr}\eta^2) \right] \qquad (3.134)$$

In terms of Kummer's function this yields to:

$$P = C_{13} + C_{14}\eta e^{\frac{d_1-\alpha}{Pr}\eta^2} 1F1[1; \frac{3}{2}; -\frac{d_1-\alpha}{2Pr}\eta^2] \qquad (3.135)$$

Integrating equation (3.135) we get the complementary solution for the stream function as follows:

$$f_c = C_{13}\eta + C_{14} 1F1[-\frac{1}{2}; \frac{1}{2}; \frac{d_1-\alpha}{2Pr}\eta^2] + C_{15} \qquad (3.136)$$

and the first derivative of this function is:

$$f_c' = C_{13} + C_{14}\eta 1F1[\frac{1}{2}; \frac{3}{2}; \frac{d_1-\alpha}{2Pr}\eta^2] \qquad (3.137)$$

The particular solution is found assuming it in the form:

$$f_p = \sum_{m=0}^{\infty} A_m \eta^{m+2} \qquad (3.138)$$

The first, second and third derivatives are as follows:

$$\left.\begin{aligned} f_p' &= \sum_{m=0}^{\infty}(m+2)A_m\eta^{m+1} \\ f_p'' &= \sum_{m=0}^{\infty}(m+2)(m+1)A_m\eta^{m} \\ f_p''' &= \sum_{m=1}^{\infty}m(m+2)(m+1)A_m\eta^{m-1} \end{aligned}\right\} \quad (3.139)$$

By substituting these terms in equation (3.127) we get:

$$\left.\begin{aligned} &6A_1 \\ &+\sum_{m=0}^{\infty}[(m+2)(m+3)(m+4)A_{(m+2)}- \\ &\tfrac{d_1-\alpha}{Pr}(m+1)(m+2)A_m]\eta^{m+1} \\ &=\sum_{n=0}^{\infty}\tfrac{(1)_n}{(\frac{3}{2})_n n!}(\tfrac{d_1-\alpha}{Pr})^n\eta^{(n+1)}-\sum_{n=0}^{\infty}(\tfrac{d_1-\alpha}{2})^n\tfrac{1}{n!}\eta^{2n} \end{aligned}\right\} \quad (3.140)$$

By comparing the coefficients of both sides of equation (3.145) one could obtain the coefficient of the particular part of the solution namely A_m.

For m = odd:

$$A_{m+2}=\frac{1}{(m+2))m+3)(m+4)}\left[\frac{d_1-\alpha}{Pr}(m+1)(m+2)A_m-(\frac{d_1-\alpha}{2})^k\frac{1}{k!}\right] \quad (3.141)$$

where, $k=\frac{m-1}{2}$

for $m = even$:

$$\left.\begin{aligned} A_m &= \tfrac{Pr}{(d_1-\alpha)(m+1)(m+2)}[(m+2)(m+3)(m+4)A_{m+2} \\ &+C_{11}\tfrac{(1)_k}{(\frac{3}{2})_k k!}(\tfrac{d_1-\alpha}{2})^k\ ] \end{aligned}\right\} \quad (3.142)$$

where, $k=\frac{m}{2}$

Thus the final solution for the unsteady laminar regime is:

$$f=C_{13}\eta+C_{14}1F1[-\frac{1}{2};\frac{1}{2};\frac{d_1-\alpha}{2Pr}\eta^2]+C_{15}+f_p \quad (3.143)$$

and the velocity is:

$$f'=C_{13}+C_{14}\eta 1F1[\frac{1}{2};\frac{3}{2};\frac{d_1-\alpha}{2Pr}\eta^2]+f_p' \quad (3.144)$$

The constants C_{13}, C_{14} and C_{15} are to be found using the boundary conditions equation (3.59):

$$\left.\begin{aligned} f(0) &= 0 = C_{14} + C_{15} \\ f'(0) &= 0 = C_{13} \\ f'(\eta_\infty) &= 0 = C_{13} + C_{14}\eta_\infty 1F1[\tfrac{1}{2}; \tfrac{3}{2}; \tfrac{d_1-\alpha}{2Pr}\eta_\infty^2] + f_p'(\eta_\infty) \end{aligned}\right\} \quad (3.145)$$

Solving the system of equation (3.145) we get:

$$\left.\begin{aligned} C_{13} &= 0 \\ C_{14} &= -\frac{f_p'(\eta_\infty)}{\eta_\infty 1F1[\frac{1}{2};\frac{3}{2};\frac{d_1-\alpha}{2Pr}\eta_\infty^2]} \\ C_{15} &= \frac{f_p'(\eta_\infty)}{\eta_\infty 1F1[\frac{1}{2};\frac{3}{2};\frac{d_1-\alpha}{2Pr}\eta_\infty^2]} \end{aligned}\right\} \quad (3.146)$$

The governing equations were solved numerically using the integral-iterative method (Appendix A). The computer programm is shown in (Appendix B). The numerical solution is to be compared with the analytical one.

Chapter 4

RESULTS AND DISCUSSIONS

4.1 Introduction

The aims of the present work,set in chapter 1, were to develop a new analytical method to predict both the temperature and velocity profiles in the Trombe wall for a constant wall temperature, power law temperature distribution in the steady state regime. The governing equations of the unsteady state case were solved, too. The obtained analytical results are compared with both numerically predicted results and with some available experimental results.

4.2 Results And Discussions

Typical analytical results for both cases of power-law variation of wall and a constant wall temperature were presented. The results will be compared with numerical and some available experimental data.

1. <u>A constant wall temperature</u>

   The temperature distribution presented by the solution of equation (3.89) for the laminar natural convection is shown in Fig.(4.1).These results are compared with the numerical solution. The figure shows good agreement between the analytical and the numerical results. Fig.(4.2) shows the distribution of the Blasius velocity profile (f') and the nondimensional stream function (f) as functions of η using both analytical and numerical methods. The analytical solution for (f) and (f') were plotted using equations (3.102) and (3.103), respectively. It can be seen that the results obtained using analytical exact solution are in good agreement with that obtained using numerical techniques. These analytical solutions are also in good agreement with some experimental data [39] and [45].

Nusselt number is calculated using the following expression:

$$Nu(x) = -\left(\frac{Gr_x}{4}\right)^{\frac{1}{4}}\theta'(0) \qquad (4.1)$$

$$Nu(x) = -C_1\left(\frac{Gr_x}{4}\right)^{\frac{1}{4}} \qquad (4.2)$$

where, C_1 is a constant as computed from equation (3.88).

An expression for Nusselt number was found experimentally to be given by [54]:

$$Nu(x) = 0.39(Gr_x Pr)^{\frac{1}{4}} \qquad (4.3)$$

However, the local Nusselt number may be expressed as:

$$Nu(x) = H(Pr)\sqrt[4]{Gr_x} \qquad (4.4)$$

where, the values of $H(Pr)$ can be obtained from a numerical solution of the governing equations. The overall dependence of $H(Pr)$ on Pr was suggested by Le Fevre [54] in terms of the empirical relationship:

$$H(Pr) = \frac{3}{4}\left[\frac{Pr}{2.43478 + 4.884Pr^{\frac{1}{2}} + 4.95283Pr}\right]^{\frac{1}{4}} \qquad (4.5)$$

The fluid considered in this study was air, for which Prandtl number $Pr = .72$.

Table (4.1) represents values for $H(Pr)$ found by analytical, numerical, and experimental methods.

Table (4.1)

	numerical	experimental	analytical
H(Pr)	0.38711	0.35925	0.36324

The error between experimental results and those obtained using the present technique is about 1% compared with 7.5% for the numerical ones.

2. The power-law distribution

The power law distribution considered at the wall is in the form:

$$T_w - T_a = Nx^n \tag{4.6}$$

the first case considered is when $n = 0$, as that of an isothermal surface,i .e the temperature at the wall is constant. This was considered in the previous section. The other physical circumstances that relate to the various values of n can be determined by studying the physical quantities such as heat flux, boundary-layer thickness, etc. The restrictions, if any, on the value of N must also be determined. For a heated wall surface, $T_w > T_a$ and N is positive.

If $n < 0$, the temperature difference $(T_w - T_a)$ becomes infinite as $x \rightarrow 0$, for a finite value of N, and decreases with x. This would be acceptable only if an infinitesimal thermal source exists at $x = 0$. For $n > 0$, the wall surface temperature is simply equal to T_a at $x = 0$ and increases with x. The heat flux $q(x)$ at a given location x may be expressed as:

$$q(x) = [-\theta'(0)]kA(x)C(x)$$

where,

$$A(x) = Nx^n \qquad C(x) = \frac{1}{x}\sqrt[4]{\frac{Gr_x}{4}} \tag{4.7}$$

Therefore,

$$q(x) \sim x^n . x^{\frac{n+3}{4}-1} = x^{\frac{5n-1}{4}} \tag{4.8}$$

the heat flux $q(x)$ varies with x as indicated in equation (4.8) for the power law temperature distribution. A particular case occurs, when $(T_w - T_a)$ varies as $x^{\frac{1}{5}}$, and the heat flux $q(x)$=const, which corresponding to uniform heat flux case. The boundary layer thickness $\delta(x)$ and the velocity u are:

$$\delta(x) \sim \frac{x}{\sqrt[4]{Gr_x}} \sim x^{\frac{1-n}{4}} \tag{4.9}$$

$$u = \nu D(x)C(x)f' \sim x^{\frac{n+1}{2}} \tag{4.10}$$

The value of n must be less than unity for the boundary layer thickness to be zero at $x = 0$, and to increase with x, which was experimentally observed [54]. Also, n must be greater than -1.0 for u to satisfy the condition that $u = 0$ at $x = 0$, which is physically expected. This gives $-1 < n < 1$.

The temperature distribution obtained using the new technique which is presented by equation (3.115) for n=0.33 is compared with the numerical results are shown in Fig.(4.3). Both results are in good agreement. Fig.(4.4) showed the non-dimensional stream function f and velocity f' profiles as a function of η for n=0.33, using numerical results and analytical solution presented by equations (3.123) and (3.124),respectively. A good agreement was found between these results.

The temperature distribution for various values of n=-0.33, -0.1, 0.0, 0.33, 0.5 are presented in Fig.(4.5). As shown in this figure at a given value of η the dimensionless temperature increased with n decreased. Fig.(4.6) shows the non-dimensional stream function f for the same values of n as indicated above. This figure indicated that f increased as the value of n is decreased. The Blasius velocity f' profiles for different n values are shown in Fig.(4.7). This figure shows that the velocity f' reaches its maximum value when $n \approx$ 1.0. This peak value of f' decreases as n increases. Fig.(4.8) presents the temperature distribution for the particular case of $n =$ 0.2 corresponds to a uniform heat flux. The velocity and stream functions for this case were shown in Fig.(4.9).

The local Nusselt number $Nu(x)$ is given by:

$$Nu(x) = H(Pr, n)Gr_x^{\frac{1}{4}} \tag{4.11}$$

The analytical expression for the Nusselt number can be estimated as follows:

$$Nu(x) = -C_7(\frac{Gr_x}{4})^{\frac{1}{4}} \tag{4.12}$$

where, C_7 is a constant obtained in equation (3.114).

The parameter H is now a function of Pr and n. For air the analytical computed values and the numerical ones obtained by [54] are presented in table (4.2).

Table (4.2)

n	H(Pr,n)	
	numerical	analytical
-.333	.310	.323
-.1	.480	.495
0	.520	.517
.333	.640	.610
.5	.66	.685

The function $Nu(x)/(Gr_x/4)^{\frac{1}{4}}$ is plotted against n in Fig.(4.10). For $n < -0.6$, the function is found to be negative, indicating that heat is transferred from the fluid to the heated surface for which $T_w > T_a$. Since $T_w > T_a$, this implies an infinite thermal source at

$x = 0.0$ and which supplies energy to the surface at all x. This is clearly physically unreasonable.

For $n = -0.6$ it was found that the function is equal to zero. This could be explained by the fact that the total amount of energy convicted by the flow in the boundary layer, $Q_c \sim x^{\frac{5n+3}{4}}$ is a constant value, which would apply for an adiabatic surface, so that no heat transfer occurs from the surface as the flow proceeds downstream.

The temperature distribution for the case when $n = -0.6$ is plotted in Fig.(4.11).

3. Unsteady laminar case

Steady laminar natural convection flow had been considered so far, in which the velocity and temperature fields do not vary with time. In some solar energy systems, the transient natural convection flow and the related transport rates are of importance. The similarity solution in terms of a similarity variable $\eta(x, y, t)$ was achieved. The condition for similarity solution to be attained was that the wall temperature distribution as a function of x and t is in the form:

$$G_w = \frac{d_8^2}{2x - 2d_1 d_8 t} \tag{4.13}$$

where,d_8 is an arbitrary constant and d_1 is a parameter.

The temperature distributions obtained by the analytical new technique presented by equation (3.131) and that using the numerical method are shown in Fig.(4.12). This figure showed that both analytical and numerical results are in good agreement. Fig.(4.13) presented the distributions of the stream function f and velocity f'. The analytical results compared well with the numerical ones.

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary of the thesis

The literature survey presented,in chapter 2, revealed that there is a lack of analytical solution for the governing equation of a Trombe wall encountered in a passive solar system. The number of analytical solutions available in the literature is very limited. This is due to the inevitable coupling between the governing equations except for extremely simplified cases.

The present work was to develop a new approximate analytical solution for some cases encountered in the Trombe wall.

5.2 Conclusions

Several points have emerged from this study which was taken to develop a new technique for finding out analytical solution for heat transfer problems, with particular emphasis to the problem of Trombe wall.

In general the new analytical technique proves to be efficient in predicting the various thermal and hydrodynamic properties in the Trombe wall.

These points con be summarized as follows:

1. A very good agreement was obtained for the temperature distribution in the channel when both analytical and numerical techniques were used. The average error was less than 0.2%.

2. The average deviation of the analytical solution from that of the numerical solution for the nondimensional stream function (f) was about 1.5%.

3. The analytical solution of Blasius velocity f' was well predicted compared with that obtained using the numerical technique. The average error was about 2.5%.

4. As was shown in table (4.1) the analytical value for the Nusselt number deviated only 1% from that of the experimental results while the numerical value deviates by 7.5%.

5. The temperature distribution for any value of n may be estimated using the present analytical solution, on the other hand using numerical solution will be limited by the computer time required to find the solution.

6. The temperature distribution for any n has a good agreement when both analytical and numerical techniques were used. The average error was less than 0.4%.

7. The average deviation of the analytical solution from the numerical solution for the stream function for any value of n was less than 2.0%.

8. The analytical solution of the velocity (f') has a good agreement when compared with the numerical solution for any n, and the average error was about 3.0%.

9. The analytical temperature distribution for unsteady case obtained using this new technique, was in a very good agreement with the numerical results. The average error was less than 1%.

10. The nondimensional stream function for the unsteady state case was well predicted compared with the numerical solution, and the error was about 4%.

11. The Blasius velocity profile obtained using new analytical technique deviated in about 5.3% from that of the numerical solution.

5.3 Recommendations

The review of the literature survey together with the forms of the governing equations solved in the present investigation, suggest that there is a room for the present new technique to be used to solve similar some problems in heat transfer and fluid mechanics.

Some of recommendations for future works may be summarized as follows:

1. The guessed function distribution (stream function) was assumed to be in the form $f = \alpha\eta$ equation (3.73), to improve the solution this function may be assumed in other forms like $f = \alpha\eta + \beta$ or $f = \alpha\eta^2$. Using these forms the resulting equations to be solved will be more complicated and need much more effort to solve.

2. The geometry of the channel in this study was taken as a vertical parallel plates. Other geometries should be investigated.

3. For flow over a wedge with free stream velocity variations of the form $U = Cx^m$, where C constant and m is related to the wedge angle $(\beta\pi)$, the similarity solution may be attained and then solved using this new technique.

4. The technique presented here may be used to investigate the problem of transpiration on a flat plate-laminar boundary layer.

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Appendix A

NUMERICAL COMPUTATION

$$f''' + 3ff'' - 2f'^2 + \theta = 0 \quad (A.1)$$

$$\theta'' + 3Prf\theta' = 0 \quad (A.2)$$

Integrating (2) we get:

$$\theta' = N_1 e^{-\int_0^\eta 3Prf d\eta} \quad (A.3)$$

$$\theta = N_1 \int_0^\eta [e^{-\int_0^\eta 3Prf d\eta}] d\eta + N_2] \quad (A.4)$$

Using the boundary conditions one could get:

$$\left.\begin{aligned} \theta(0) &= 1 = N_2 \\ \theta(\eta_\infty) &= 0 = 1 + N_1 \int_0^{\eta_\infty} [e^{-\int_0^\eta 3Prf d\eta}] d\eta \end{aligned}\right\} \quad (A.5)$$

$$N_1 = -\frac{1}{\int_0^{\eta_\infty} [e^{-\int_0^\eta 3Prf d\eta}] d\eta} \quad (A.6)$$

Finally the temperature distribution is:

$$\theta = 1 - \frac{\int_0^\eta [e^{-\int_0^\eta 3Prf d\eta}] d\eta}{\int_0^{\eta_\infty} [e^{-\int_0^\eta 3Prf d\eta}] d\eta} \quad (A.7)$$

Rearrange equation ($A.1$) in the form:

$$f''' + 3ff'' = 2f'^2 - \theta \quad (A.8)$$

Integrating this equation we get:

$$f'' = e^{-\int_0^\eta 3f d\eta} [\int_0^\eta [e^{\int_0^\eta 3f d\eta} (2f'^2 - \theta)] d\eta + N_3] \quad (A.9)$$

$$
\left.\begin{aligned}
f' &= \int_0^\eta f''\,d\eta \\
&= \int_0^\eta \{e^{-\int_0^\eta 3f d\eta}\left[\int_0^\eta [e^{\int_0^\eta 3f d\eta}(2f'^2-\theta)]d\eta + N_3\right]\}d\eta + N_4
\end{aligned}\right\} \quad (A.10)
$$

With the boundary conditions:

$$
f'(0) = 0 \qquad \Rightarrow \qquad N_4 = 0 \qquad (A.11)
$$

$$
\left.\begin{aligned}
f &= \int_0^\eta f'\,d\eta \\
&= \int_0^\eta \left\{\int_0^\eta \left(e^{-\int_0^\eta 3f d\eta}\right.\right. \\
&\left.\left.\left\{\int_0^\eta \left[e^{\int_0^\eta 3f d\eta}\left(2f''-\theta\right)\right] d\eta + N_3\right\}\right) d\eta\right\} d\eta + N_5
\end{aligned}\right\} \quad (A.12)
$$

The boundary conditions are:

$$
f(0) = 0 \qquad \Rightarrow \qquad N_5 = 0 \qquad (A.13)
$$

$$
\left.\begin{aligned}
f^{\infty} &= 0 \\
N_3 &= -\frac{\int_0^\eta \left\{e^{-\int_0^\eta 3f d\eta} \int_0^\eta \left[e^{\int_0^\eta 3f d\eta}\left(2f'^2-\theta\right)\right] d\eta\right\} d\eta}{e^{-\int_0^\infty 3f d\eta} d\eta}
\end{aligned}\right\} \quad (A.14)
$$

The numerical method of solution is described as follows:

1. Assume any distribution for f'
2. Calculate f using any integrating scheme (the Simpson'$\frac{1}{3}$ Rule method was used)
3. From equation $(A.7)$ calculate θ
4. Using Equation $(A.10)$ calculate f'.
5. If $f'_{last} = f'_{previous} \Rightarrow$ STOP.
6. ELSE GOTO STEP 2.

Appendix B

PROGRAMME LISTING

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C           A PROGRAMME FOR SOLVING NONLINEAR EQUATIONS                C
C                    FOR HEAT TRANSFER PROBLEM                         C
C                                                                      C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C        THE EQUATIONS ARE:
C        f'''+3.*f*f''-2*f'**2+Q=0 ................(1)
C        Q''+3.*PR*f*Q'=0 .........................(2)
C        TO SOLVE THESE EQUATIONS WE WILL USE AN
C        ITERATIVE INTEGRAL METHOD
C        F(X)=THE GUESSED FUNCTION
C        F1(X)=THE FIRST DERIVATIVE"f'"
C        S(X)=THE FUNCTION "f"
C        S1(X)=3.*PR*S(X)
C        S2(X)=INTEGRAL OF (S1(X))
C        S3(X)=EXP(-S2(X))
C        S4(X)=INTEGRAL OF (S3(X))
C        S5(X)=3.*S(X)
C        S6(X)=INTEGRAL OF (S5(X))
C        S7(X)=EXP(S6(X))
C        S8(X)=2.*F1(X)**2-THETA(X)
C        S9(X)=S8(X)*S7(X)
C        S10(X)=INTEGRAL OF (S9(X))
C        S11(X)=EXP(-S6(I))
C        S12(X)=S10(X)+C3
C        S13(X)=S11(X)*S12(X)
C        S14(X)=INTEGRAL OF (S13(X))
C        S15(X)=S10(X)*S11(X)
C        S16(X)=INTEGRAL OF (S15(X))
C        S17(X)=INTEGRAL OF (S11(X))
C        C3=-S16(BB)/S17(BB)
C        THETA=THE TEMPERATURE DISTRIBUTION
C        THETA=1-S4(X)/S4(BB)
C        B=THE LAST X
      PARAMETER(EN=1./3.,PR=.72,N=401,H=.011,D1=2.E-7,ERR=555)
      DIMENSION F1(500),S(500),S1(500),S2(500),THETA(500)
    &,  S3(500),S4(500),S5(500),S6(500),S7(500),V(500),WWE(500)
    &,  S8(500),S9(500),S10(500),S11(500),S12(500),S24(500)
    &,  SUM(500),SUM2(500),SUM1(500),S22(500),S23(500),WE(500)
```

```
     &,  S13(500),S14(500),S15(500),S16(500),S17(500),R(500)
     &,  X(500),F(500),FF(500),S18(500),S19(500),S20(500),S21(500)
         OPEN(5,file='SAD1.OUT',STATUS='OLD')
         OPEN(6,file='SAD.OUT',STATUS='NEW')
         DO 33 I=1,N
         READ(5,222)X(I),S(I),F1(I),THETA(I)
33       CONTINUE
C        GUESS ANY DISTRIBUTION OF f'SSUCH THAT SATISFIES THE
C        BOUNDARY CONDITIONS.
2876     DO 444 I=1,N
         S13(I)=(EN+3.)*PR*S(I)
444      CONTINUE
         DO 12 I=1,N
         V(I)=S13(I)
12       CONTINUE
         CALL SIMPS(V,R)
         DO 16 I=1,N
         S14(I)=R(I)
16       CONTINUE
         DO 50 I=1,N
         S15(I)=EXP(-S14(I))
50       CONTINUE
         DO 51 I=1,N
         S16(I)=EXP(S14(I))
51       CONTINUE
         DO 80 I=1,N
         S17(I)=4.*EN*PR*F1(I)*THETA(I)
80       CONTINUE
         DO 190 I=1,N
         S18(I)=S17(I)*S16(I)
190      CONTINUE
         DO 110 I=1,N
         V(I)=S18(I)
110      CONTINUE
         CALL SIMPS(V,R)
         DO 123 I=1,N
         S19(I)=R(I)
123      CONTINUE
         DO 742 I=1,N
         S20(I)=S19(I)*S15(I)
742      CONTINUE
         DO 130 I=1,N
         V(I)=S20(I)
130      CONTINUE
         CALL SIMPS(V,R)
         DO 1123 I=1,N
         S21(I)=R(I)
1123     CONTINUE
         DO 137 I=1,N
         V(I)=S15(I)
137      CONTINUE
         CALL SIMPS(V,R)
         DO 1129 I=1,N
         S22(I)=R(I)
1129     CONTINUE
         CC1=-(1+S21(N))/S22(N)
         DO 140 I=1,N
         S23(I)=S21(I)+CC1*S22(I)+1
140      CONTINUE
         S24(1)=S15(1)*S19(1)+CC1*S15(1)
```

```
      DO 5243 I=1,N
      WE(I)=ABS(S23(I)-THETA(I))
5243  CONTINUE
      DO 4253 I=1,N
      IF(WE(I).LE.D1) THEN
      CONTINUE
      IF(I.EQ.N) GO TO 919
      ELSE
      GO TO 446
      ENDIF
4253  CONTINUE
446   DO 176 I=1,N
      THETA(I)=ABS(S23(I))
176   CONTINUE
      GO TO 2876
919   DO 534 I=1,N
      V(I)=F1(I)
534   CONTINUE
      CALL SIMPS(V,R)
      DO 656 I=1,N
      S(I)=R(I)
656   CONTINUE
      DO 122 I=1,N
122   S1(I)=(EN+3.)*S(I)
      DO 1163 I=1,N
      V(I)=S1(I)
1163  CONTINUE
      CALL SIMPS(V,R)
      DO 1161 I=1,N
      S2(I)=R(I)
1161  CONTINUE
      DO 2123 I=1,N
2123  S3(I)=EXP(-S2(I))
      DO 124 I=1,N
124   S4(I)=EXP(S2(I))
      DO 1263 I=1,N
      V(I)=S3(I)
1263  CONTINUE
      CALL SIMPS(V,R)
      DO 1162 I=1,N
      S5(I)=R(I)
1162  CONTINUE
      DO 1125 I=1,N
      S6(I)=2.*(EN+1.)*F1(I)**2-S23(I)
1125  CONTINUE
      DO 126 I=1,N
126   S7(I)=S6(I)*S4(I)
      DO 169 I=1,N
      V(I)=S7(I)
169   CONTINUE
      CALL SIMPS(V,R)
      DO 167 I=1,N
      S8(I)=R(I)
167   CONTINUE
      DO 521 I=1,N
      S9(I)=S8(I)*S3(I)
521   CONTINUE
      DO 643 I=1,N
      V(I)=S9(I)
643   CONTINUE
```

```
      CALL SIMPS(V,R)
      DO 177 I=1,N
      S10(I)=R(I)
177   CONTINUE
      C11=-S10(N)/S5(N)
      DO 499 I=1,N
      S11(I)=S10(I)+C11*S5(I)
499   CONTINUE
      DO 5143 I=1,N
      WWE(I)=ABS(S11(I)-F1(I))
5143  CONTINUE
      DO 4153 I=1,N
      IF(WWE(I).LE.D1) THEN
      CONTINUE
      IF(I.EQ.N) GO TO 789
      ELSE
      GO TO 1446
      ENDIF
4153  CONTINUE
1446  DO 1176 I=1,N
      F1(I)=ABS(S11(I))
1176  CONTINUE
      GO TO 2876

789   DO 223 I=1,N
223   WRITE(6,222) X(I),S(I),F1(I),THETA(I)
222   FORMAT(2X,'X(I)=',D20.13,2X,'S(I)=',D20.13,2X,'F1(I)='
   &, D20.13,2X,'THETA(I)=',D20.13)
555   STOP
      END
```

SUBROUTINE SIMPS(V,R)

```
      PARAMETER(H=.011,N=401)
      DIMENSION SUM2(500),SUM(500),Y(500),NM(500)
      DIMENSION DK(500),SUM1(500),V(500),R(500)
      SUM(1)=0.
      SUM(2)=H/2.*(V(1)+V(2))
      SUM3=0.
      SUM4=0.
      DO 221 K=3,N
      NM(K)=K/2
      Y(K)=K/2.
      DK(K)=Y(K)-NM(K)
      IF(DK(K).EQ.0) THEN
      SUM(K)=(V(1)+V(K))*H/3.
      SUM1(K)=SUM3+H/3.*(2.*V(K-1))
      SUM3=SUM1(K)
      SUM(K)=SUM(K)+SUM3+SUM4
```

```
      PARAMETER(B=1.5,ALFA=.1900624,PR=.72,N=401,N1=33,H=.011,PI=22./7.
     &,    BD=.5,NN=57,NN1=56,A10=-4.095125675201,EN1=0.
     &,    TT=-.5137015581131)
      double precision AA,CC,BB,TERM
      DIMENSION ADD(70),F(500),X(500),AA1(70),AA11(70),F2(500),FS1(500)
      DIMENSION SUM1(500),THETA(500),F1(500),SUM2(500),FF(500)
      OPEN(4,FILE='NAT.OUT',STATUS='NEW')
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
      C10=-A10
      A=-(EN1+1.)/(EN1+3)
      AD=-(3.*EN1+5.)/(2.*(EN1+3.))
      B10=-1./2./ALFA
      DO 110 I=1,N
      X(I)=H*FLOAT(I-1)
      SUM=1.0
      DO 920 IK=1,N1
      Y=((EN1+3.)*ALFA/2.)**.5*X(I)
      AA=1.
      BB=1.
      CC=1.
      DO 49 K=1,IK
      AA=AA*(A+K-1)
      BB=BB*(B+K-1)
      CC=CC*FLOAT(K)
49    CONTINUE
      M=2*IK
      TERM=(AA/BB)/CC*Y**M
      SUM=SUM+(-1)**IK*TERM
920   CONTINUE
      SS=1.
      SUM1(I)=SUM
      DO 9920 IK=1,N1
      Y=((EN1+3.)*ALFA/2.)**.5*X(I)
      AA=1.
      BB=1.
      CC=1.
      DO 94 K=1,IK
      AA=AA*(AD+K-1)
      BB=BB*(BD+K-1)
      CC=CC*FLOAT(K)
94    CONTINUE
      M=2*IK
      TERM=(AA/BB)/CC*Y**M
      SS=SS+(-1)**IK*TERM
9920  CONTINUE
      SUM2(I)=SS
      BETA=((EN1+3.)*ALFA*PR/2.)
      AZ=3.*(1-EN1)/2./(EN1+3.)
      BA=1.5
      O=0.
      SW=1./2./ALFA*X(I)
      DO 4 K=NN,1,-2
      BK=(K+1)*((K-2)*EN1+3.*K-2.)
      AA=1.
      BB=1.
      RI=1.
      KP=(K+3)*(K+2)*(K+1)
```

```
      AMI=FLOAT(K-1)/2.
      KJ=AMI+1
      DO 5 II=1,AMI
      AA=AA*(AZ+II-1)
      BB=BB*(BA+II-1)
      RM=RI*FLOAT(II)
      RI=RM
 5    CONTINUE
      AA1(K)=1./(BK*ALFA)*((-1)**KJ*BETA**AMI/BB/RI*AA*TT-KP*O)
      O=AA1(K)
      F(I)=SW+AA1(K)*X(I)**(K+1)
      SW=F(I)
 4    CONTINUE
      A1=-2.*EN1/(EN1+3.)
      B1=.5
      S1=0.
      O1=0.
      DO 45 II=NN1,1,-2
      BK1=FLOAT(II*(II+1))*(EN1+3.)-2.*FLOAT(II+1)*(EN1+1)
      AA2=1.
      BB2=1.
      RI2=1.
      KP2=(II+3)*(II+2)*(II+1)
      AMI2=FLOAT(II)/2.
      KJ1=AMI2+1
      DO 55 III=1,AMI2
      AA2=AA2*(A1+III-1)
      BB2=BB2*(B1+III-1)
      RM2=RI2*FLOAT(III)
      RI2=RM2
 55   CONTINUE
      AA11(II)=1./(BK1*ALFA)*((-1)**KJ1*BETA**AMI2/BB2/RI2*AA2*TT-KP2*O1
      O1=AA11(II)
      FS1(I)=S1+AA11(II)*X(K)**(II+1)
      S1=FS1(I)
 45   CONTINUE
      F2(I)=F(I)+FS1(I)
      FF(I)=B10*SUM1(I)*X(I)+A10*SUM2(I)+C10+F2(I)
 110  CONTINUE
      PRINT*,F2(N)
      PRINT*, SUM2(N)     ,SUM1(N)    ,  FF(N),  F(N)
      DO 90 I=1,N
 90   WRITE(4,67)X(I),FF(I)
 67   FORMAT(2X,'X(I)=',D20.13,2X,'FF(I)=',D20.13)
 222  FORMAT(2X,'X(I)=',D20.13,2X,'S(I)=',D20.13,2X,'F1(I)=',D20.13
    &,2X,'THETA(I)=',D20.13)


      CALL INIPLT(0,.TRUE.,1.0)
      CALL VIEWPORT(0,8000,0,20000)
      CALL GRAPHBOUNDARY(2000,6800,3000,8000)
      CALL SETLEGEND(1300,1800,0)
      CALL SCALE(0.0,4.5,0.0,.8)
      CALL AXIS (1.0,'10.1','ETA',3,.20,'10.1','VELOCITY ',3)
C                (X,Y,N,COL,SYM 0-8 ,SIZE 0-9,NUM 1- ,LINE 0-8)
C     CALL POLYLINE(X,SUM1,401,0,0,0,0,0)
      CALL POLYLINE(X,FF,401,0,0,0,0,8)
C     CALL POLYLINE(X,SUM2,401,0,0,0,0,2)
C                *********************************
      CALL WRITELEGEND('Fig.1 Temperature distribution    ',0,0,2,0,9)
```

```
      CALL WRITELEGEND('                                        ',0,0,2,0,9)
      CALL WRITELEGEND('                                        ',0,0,2,0,9)
      CALL WRITELEGEND('                                        ',0,0,2,0,9)
      CALL WRITELEGEND('                                        ',0,0,2,0,9)
      CALL SETLEGEND(2000,9900,200)
C     Start writing from the bottom
C                     ********************************** */*
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('                                        ',0,0,1,0,9)
      CALL WRITELEGEND('VELOCITY                                ',0,0,0,0,0)
      CALL WRITELEGEND('VELOCITY                                ',0,0,0,0,8)
      CALL WRITELEGEND('                                        ',0,0,0,0,2)
      CALL ENDPLT
      END
```

```
      CALL VIEWPORT(0,8000,0,20000)
      CALL GRAPHBOUNDARY(2000,6800,3000,8000)
      CALL SETLEGEND(1300,1800,0)
      CALL SCALE(0.0,4.5,0.0,1.0)
      CALL AXIS (1.0,'10.1','.',3,.2,'10.1','. ',3)
C                    (X,Y,N,COL,SYM 0-8 ,SIZE 0-9,NUM 1- ,LINE 0-8)
      CALL SMOOTH(X,THETA1,401,0,0,0,0,0)
      CALL POLYLINE(X,THETA,401,0,0,0,0,2)
C     CALL POLYLINE(X,SUM2,401,0,0,0,0,2)
CC                       *********************************
      CALL WRITELEGEND('Fig. 4.1  Temperature distributions
     & ',0,0,2,0,9)
      CALL WRITELEGEND('                                    ',0,0,2,0,9)
      CALL WRITELEGEND('                                    ',0,0,2,0,9)
      CALL WRITELEGEND('                                    ',0,0,2,0,9)
      CALL WRITELEGEND('                                    ',0,0,2,0,9)
      CALL SETLEGEND(3500,8700,200)
C     Start writing from the bottom
C                        *********************************      */*
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL WRITELEGEND('Analytical solution                 ',0,0,2,0,0)
      CALL WRITELEGEND('Numerical solution                  ',0,0,2,0,2)
      CALL WRITELEGEND('                                    ',0,0,1,0,9)
      CALL ENDPLT
      END
```

```
      PARAMETER(EN1=0.,B=.5,ALFA=.1900624,PR=.72,N=401,N1=33,
     &    H=.011,PI=22./7.,BD=1.5,MM=57,A11=.5137015581131)
      double precision AA,CC,BB,TERM
      DIMENSION THETA1(500),X(500), SUM1(500),SUM2(500),S(500)
     &,   F1(500),THETA(500)
      OPEN(5,file='BAZ1.OUT',STATUS='OLD')
      OPEN(6,FILE='THETA1.OUT',STATUS='NEW')
      READ(*,*) YE
      DO 33 I=1,N
      READ(5,222)X(I),S(I),F1(I),THETA(I)
33    CONTINUE
      DO 10 I=1,N
      A=-2.*EN1/(EN1+3.)
      X(I)=H*FLOAT(I-1)
      SUM=1.0
      DO 20 IK=1,N1
      Y=((EN1+3.)*PR*ALFA/2.)**.5*X(I)
      AA=1.
      BB=1.
      CC=1.
      DO 4 K=1,IK
      AA=AA*(A+K-1)
      BB=BB*(B+K-1)
      CC=CC*FLOAT(K)
4     CONTINUE
      M=2*IK
      TERM=(AA/BB)/CC*Y**M
      SUM=SUM+(-1)**IK*TERM
20    CONTINUE
      SUM1(I)=SUM
      AD=3.*(1.-EN1)/(2.*(EN1+3.))
      X(I)=H*FLOAT(I-1)
      SUM=1.0
      DO 21 IK=1,N1
      Y=((EN1+3.)*PR*ALFA/2.)**.5*X(I)
      AA=1.
      BB=1.
      CC=1.
      DO 14 K=1,IK
      AA=AA*(AD+K-1)
      BB=BB*(BD+K-1)
      CC=CC*FLOAT(K)
14    CONTINUE
      M=2*IK
      TERM=(AA/BB)/CC*Y**M
      SUM=SUM+(-1)**IK*TERM
21    CONTINUE
      SUM2(I)=SUM
      THETA1(I)=SUM1(I)-A11*X(I)*SUM2(I)
10    CONTINUE
      DO 90 I=1,N
90    WRITE(6,67)X(I),THETA1(I)
67    FORMAT(2X,'X(I)=',D20.13,2X,'THETA1(I)=',D20.13)
222   FORMAT(2X,'X(I)=',D20.13,2X,'S(I)=',D20.13,2X,'F1(I)='
     &,   D20.13,2X,'THETA(I)=',D20.13)

      CALL INIPLT(0,.TRUE.,1.0)
```

Appendix C

TRANSFORMATIONS

$$\theta = \frac{2}{\sqrt{\pi\eta}} e^{-\frac{3\alpha Pr}{4}\eta^2} \left[C_1' M_{-\frac{1}{4},\frac{1}{4}}(\frac{3\alpha Pr}{2}\eta^2) + C_2' M_{-\frac{1}{4},-\frac{1}{4}}(\frac{3\alpha Pr}{2}\eta^2) \right] \quad (C.1)$$

$$\left.\begin{aligned} \theta = \tfrac{2}{\sqrt{\pi\eta}} e^{-\frac{3\alpha Pr}{4}\eta^2} \Big[C_1' e^{-\frac{3\alpha Pr}{4}\eta^2} (\tfrac{3\alpha Pr}{2}\eta^2)^{\frac{3}{4}} 1F1[\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{4}; 1+\tfrac{2}{4}; \tfrac{3\alpha Pr}{2}\eta^2 \\ + C_2' e^{-\frac{3\alpha Pr}{4}\eta^2} (\tfrac{3\alpha Pr}{2}\eta^2)^{\frac{1}{4}} 1F1[\tfrac{1}{2}-\tfrac{1}{4}+\tfrac{1}{4}; 1-\tfrac{2}{4}; \tfrac{3\alpha Pr}{2}\eta^2]\Big] \end{aligned}\right\} \quad (C.2)$$

$$\theta = C_1'' e^{-\frac{3\alpha Pr}{2}\eta^2} 1F1[1; \frac{3}{2}; \frac{3\alpha Pr}{2}\eta^2] + C_2'' e^{-\frac{3\alpha Pr}{2}\eta^2} 1F1[\frac{1}{2}; \frac{3}{2}; \frac{3\alpha Pr}{2}\eta^2] \quad (C.3)$$

$$\theta = C_1 + C_2 1F1[\frac{1}{2}; \frac{3}{2}; \frac{3\alpha Pr}{2}\eta^2] \quad (C.4)$$

With the boundary conditions:

$$\theta(0) = 1 \qquad \theta(\eta_\infty) = 0 \quad (C.5)$$

Thus:

$$\left.\begin{aligned} C_1 &= 1 \\ C_2 &= -\frac{1}{\eta_\infty 1F1[\frac{1}{2}; \frac{3}{2}; \frac{3\alpha Pr}{2}\eta_\infty^2]} \end{aligned}\right\} \quad (C.6)$$

Finally the temperature distribution is:

$$\theta = 1 - \frac{\eta}{\eta_\infty 1F1[\frac{1}{2}; \frac{3}{2}; \frac{3\alpha Pr}{2}\eta_\infty^2]} 1F1[\frac{1}{2}; \frac{3}{2}; \frac{3\alpha Pr}{2}\eta^2] \quad (C.7)$$

The governing equations are reduced as follows:

- **continuity**

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0 \qquad \text{(C.8)}$$

- **momentum**

$$\left.\begin{array}{l} 4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f' x^{\frac{1}{2}} \left[2x^{-\frac{1}{2}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f' \right. \\ \left. -yx^{-\frac{3}{4}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^3} f''\right] + \left(yx^{-\frac{1}{2}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f'' \right. \\ \left. -3\sqrt{\nu}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)} x^{-\frac{1}{4}} f\right) 4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^3} \\ = g\beta(T-T_a) + \frac{g\beta}{\nu}(T_w-T_a)f''' \end{array}\right\} \qquad \text{(C.9)}$$

$$\left.\begin{array}{l} 4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f' x^{\frac{1}{2}} 2x^{-\frac{1}{2}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f' \\ -4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f' x^{\frac{1}{2}} yx^{-\frac{3}{4}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^3} f'' \\ +4\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^3} f' x^{\frac{1}{2}} yx^{-\frac{1}{2}}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^2} f'' \\ -12\sqrt{\nu}\sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)} x^{-\frac{1}{4}} f \sqrt[4]{\left(\frac{g\beta(T_w-T_a)}{4}\right)^3} f'' \\ = g\beta(T-T_a) + \nu\frac{g\beta}{\nu}(T_w-T_a)f''' \end{array}\right\} \qquad \text{(C.10)}$$

Finally:

$$f''' + 3ff'' - 2f'^2 + \theta = 0 \qquad \text{(C.11)}$$

The integrating of Kummer's function.

$$P_c = C_3'' \eta e^{-\frac{3\alpha}{2}\eta^2} 1F1[\frac{4}{3};\frac{3}{2};\frac{3\alpha}{2}\eta^2] + C_4'' e^{-\frac{3\alpha}{2}\eta^2} 1F1[\frac{5}{6};\frac{1}{2};\frac{3\alpha}{2}\eta^2] \tag{C.12}$$

$$f_c = \int_0^\eta P_c d\eta \tag{C.13}$$

$$\left.\begin{aligned} f_{c1} &= \int e^{-\frac{n+3}{2}\alpha\eta^2} 1F1[\tfrac{3n+5}{2(n+3)};\tfrac{1}{2};\tfrac{n+3}{2}\alpha\eta^2]d\eta \\ &= \begin{bmatrix} y = \frac{n+3}{2}\alpha\eta^2 & dy = (n+3)\alpha\eta d\eta \\ \eta = \sqrt{\frac{2y}{(n+3)\alpha}} & d\eta = \frac{1}{(n+3)\alpha\eta}dy \end{bmatrix} \\ &= C\int e^{-y}y^{-\frac{1}{2}} 1F1[\tfrac{3n+5}{2(n+3)};\tfrac{1}{2};y]dy \\ &= Ce^{-y}y^{\frac{1}{2}} 1F1[\tfrac{3n+5}{2(n+3)};\tfrac{1}{2};y] \end{aligned}\right\} \tag{C.14}$$

Finally:

$$f_{c1} = C_1 e^{-\frac{n+3}{2}\alpha\eta^2} 1F1[\frac{5n+11}{2(n+3)};\frac{3}{2};\frac{n+3}{2}\alpha\eta^2] \tag{C.15}$$

The second term of this integral is:

$$\left.\begin{aligned} f_{c2} &= \int e^{-\frac{n+3}{2}\alpha\eta^2} \eta 1F1[2\tfrac{n+2}{n+3};\tfrac{3}{2};\tfrac{n+3}{2}\alpha\eta^2]d\eta \\ &= \begin{bmatrix} y = \frac{n+3}{2}\alpha\eta^2 & dy = (n+3)\alpha\eta d\eta \\ \eta = \sqrt{\frac{2y}{(n+3)\alpha}} & d\eta = \frac{1}{(n+3)\alpha\eta}dy \end{bmatrix} \\ &= C\int e^{-y} 1F1[2\tfrac{n+2}{n+3};\tfrac{3}{2};y]dy \\ &= Ce^{-y} 1F1[2\tfrac{n+2}{n+3};\tfrac{1}{2};y] \end{aligned}\right\} \tag{C.16}$$

And the second term is:

$$f_{c2} = C_2 e^{-\frac{n+3}{2}\alpha\eta^2} 1F1[2\frac{n+2}{n+3};\frac{1}{2};\frac{n+3}{2}\alpha\eta^2] \tag{C.17}$$

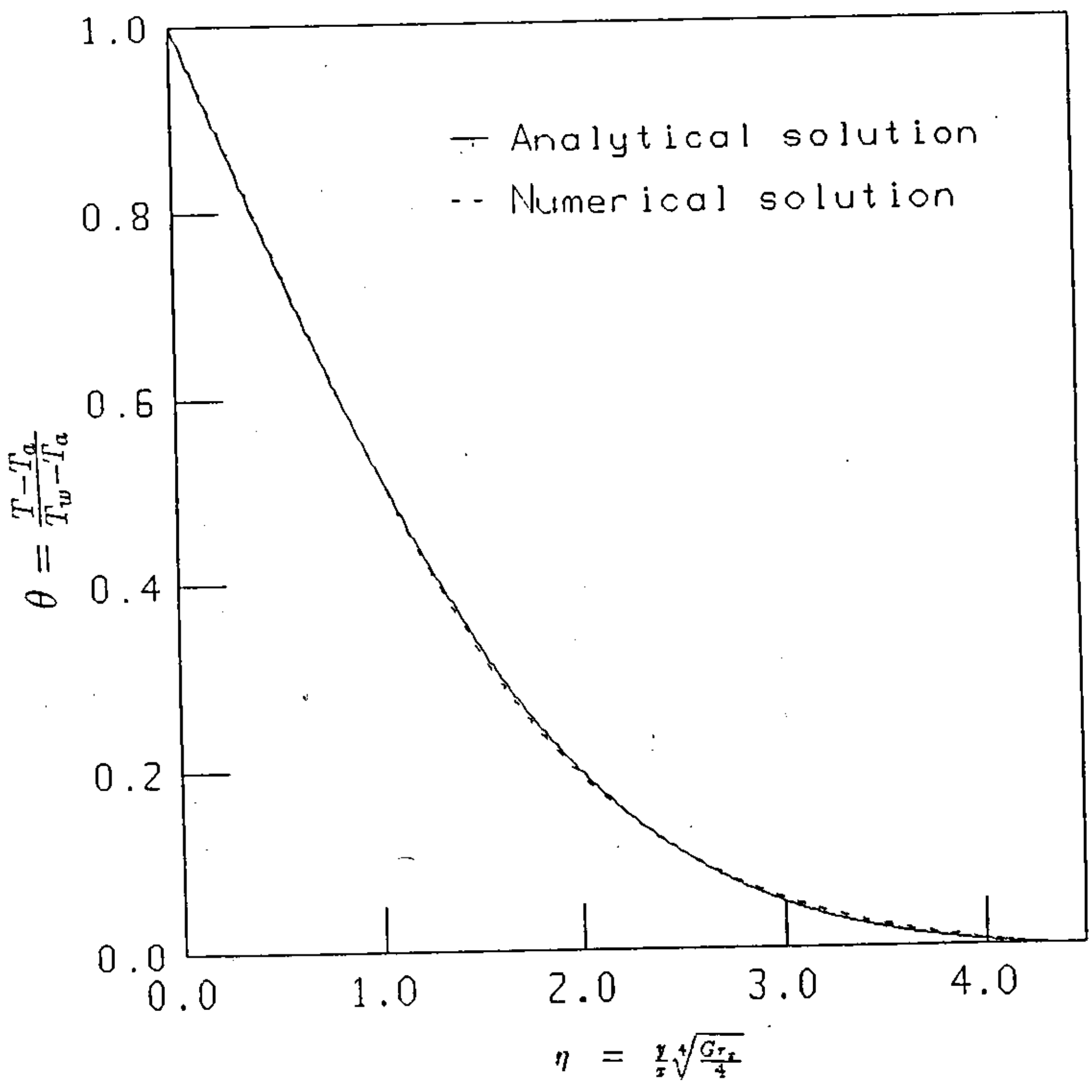


Fig. 4.1 Temperature distributions

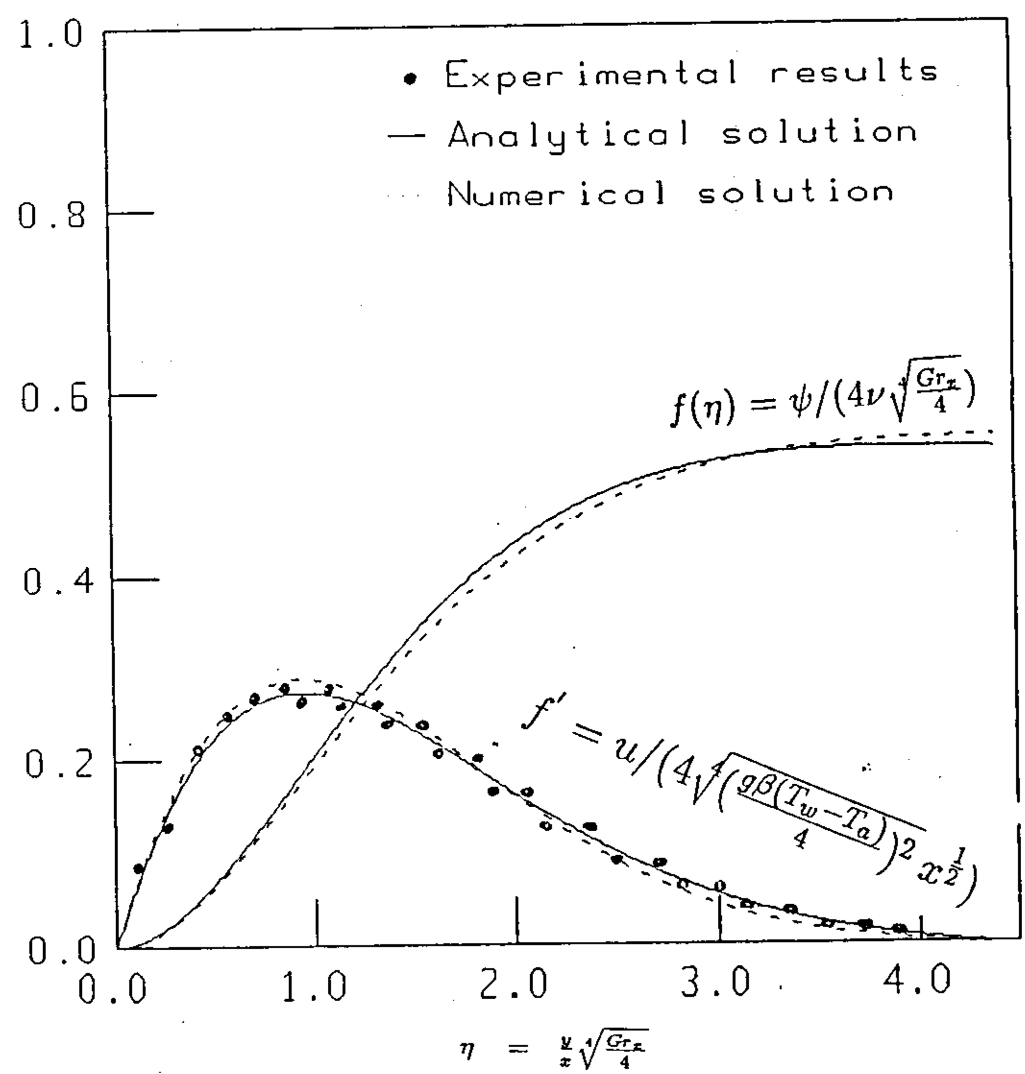


Fig.4.2 The stream and velocity profiles

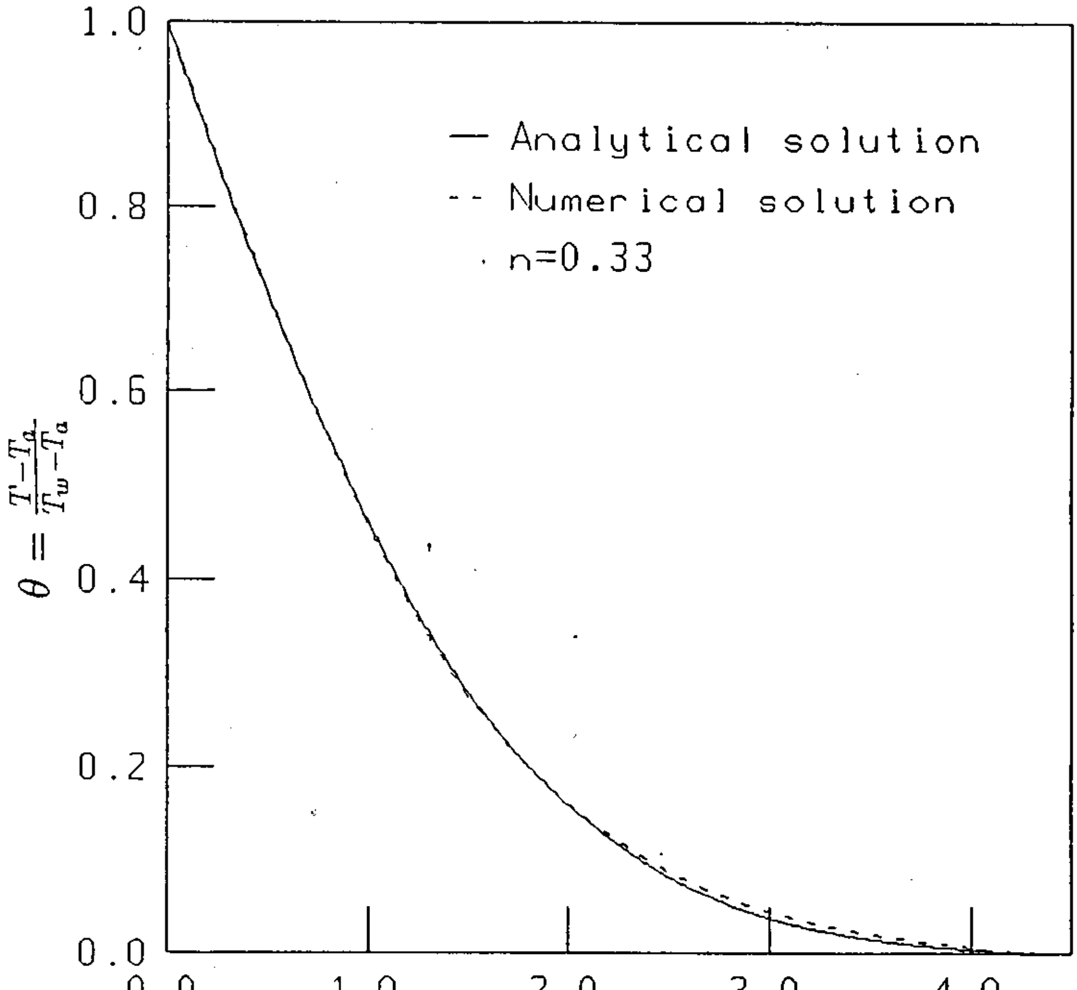


Fig.4.3 Temperature distributions

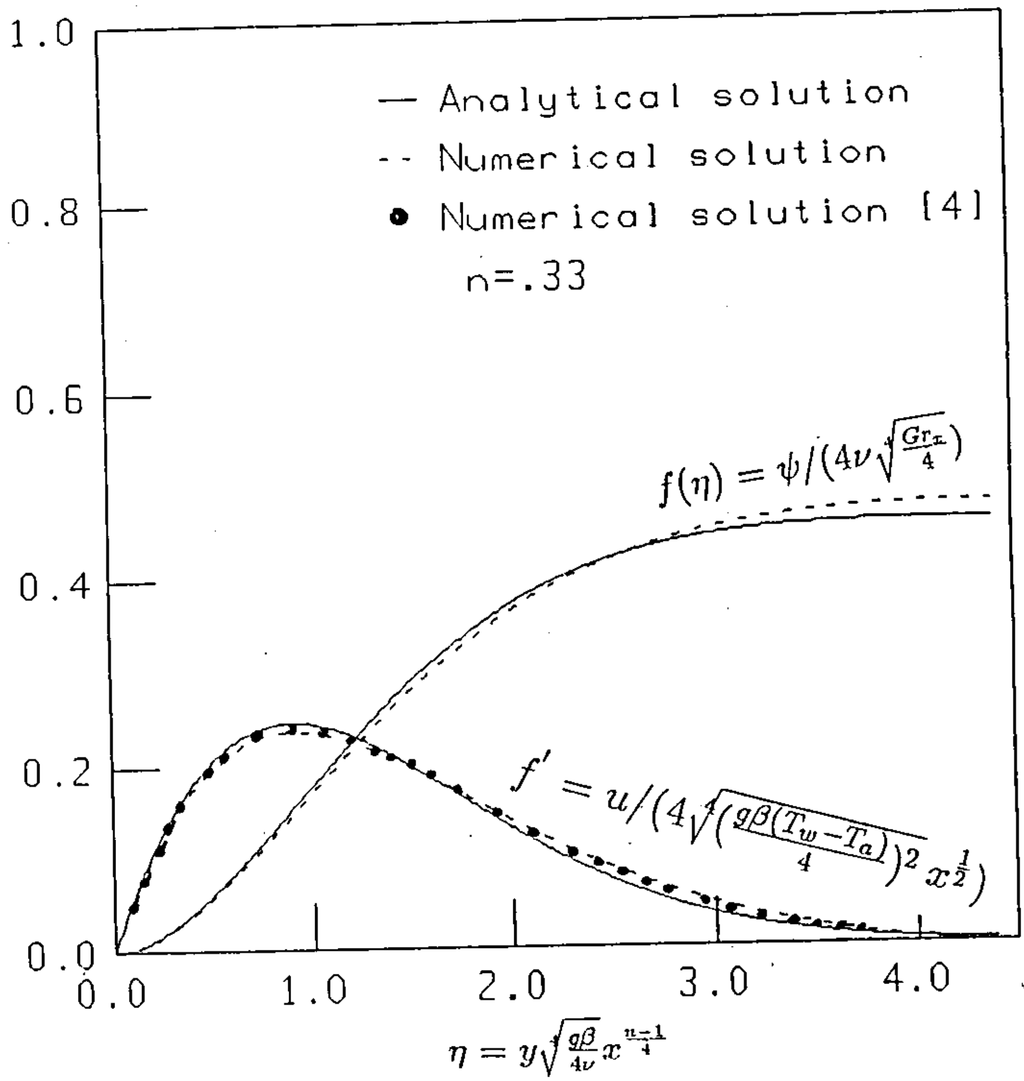
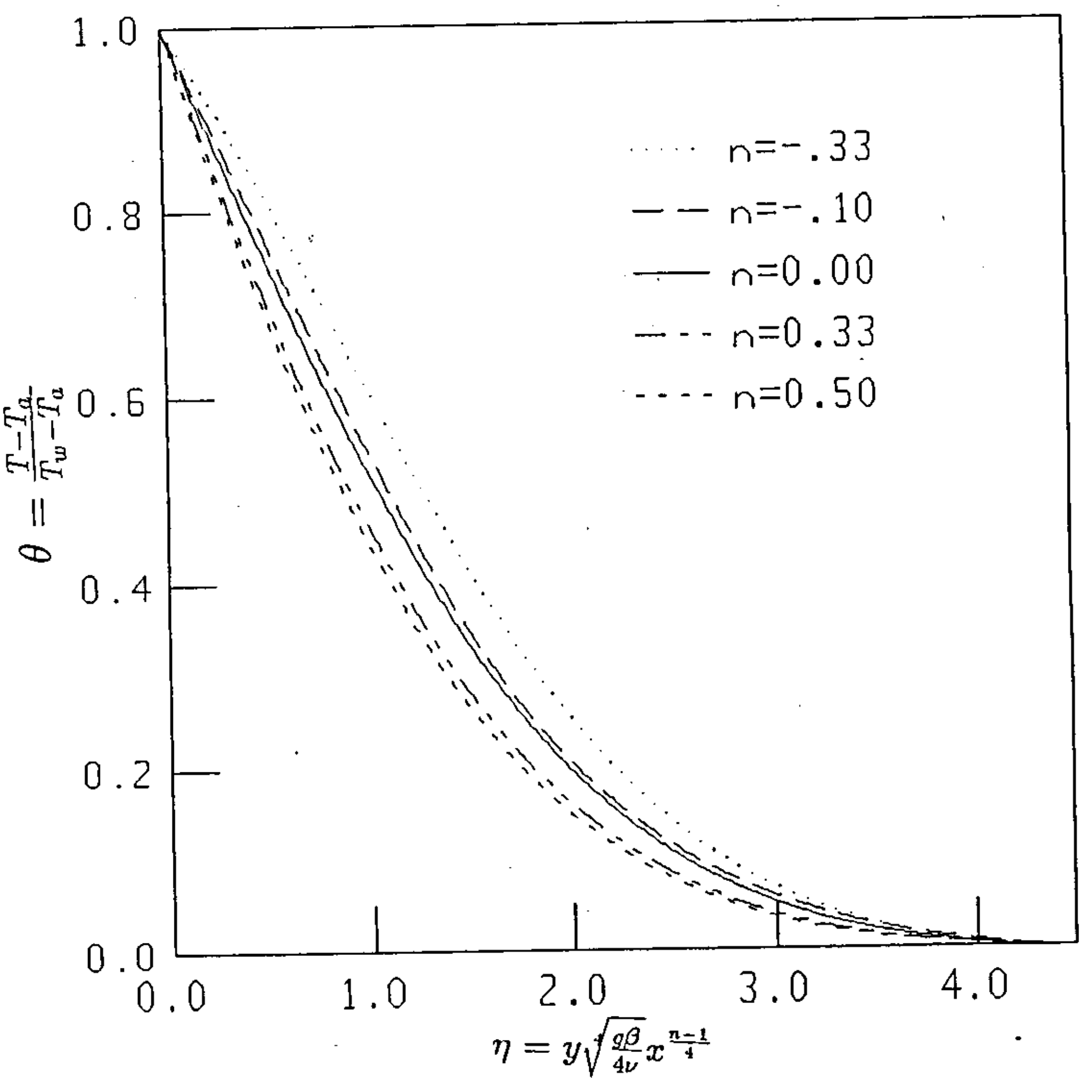


Fig.4.4 The stream function and velocity profiles



.4.5 The analytical temperature distributions for different values of n

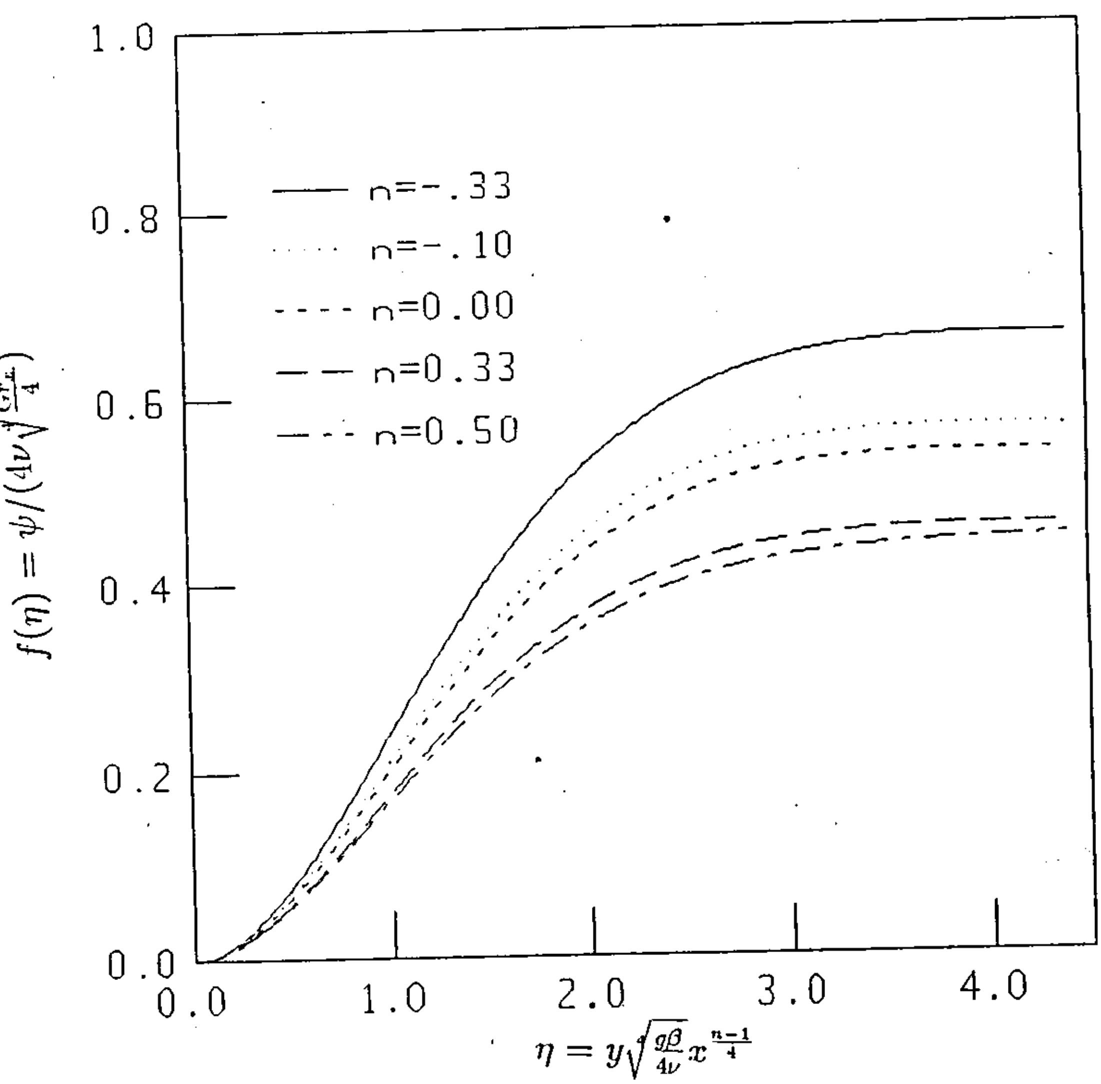


Fig.(4.6) The analytical solutions
for the stream function
for different n

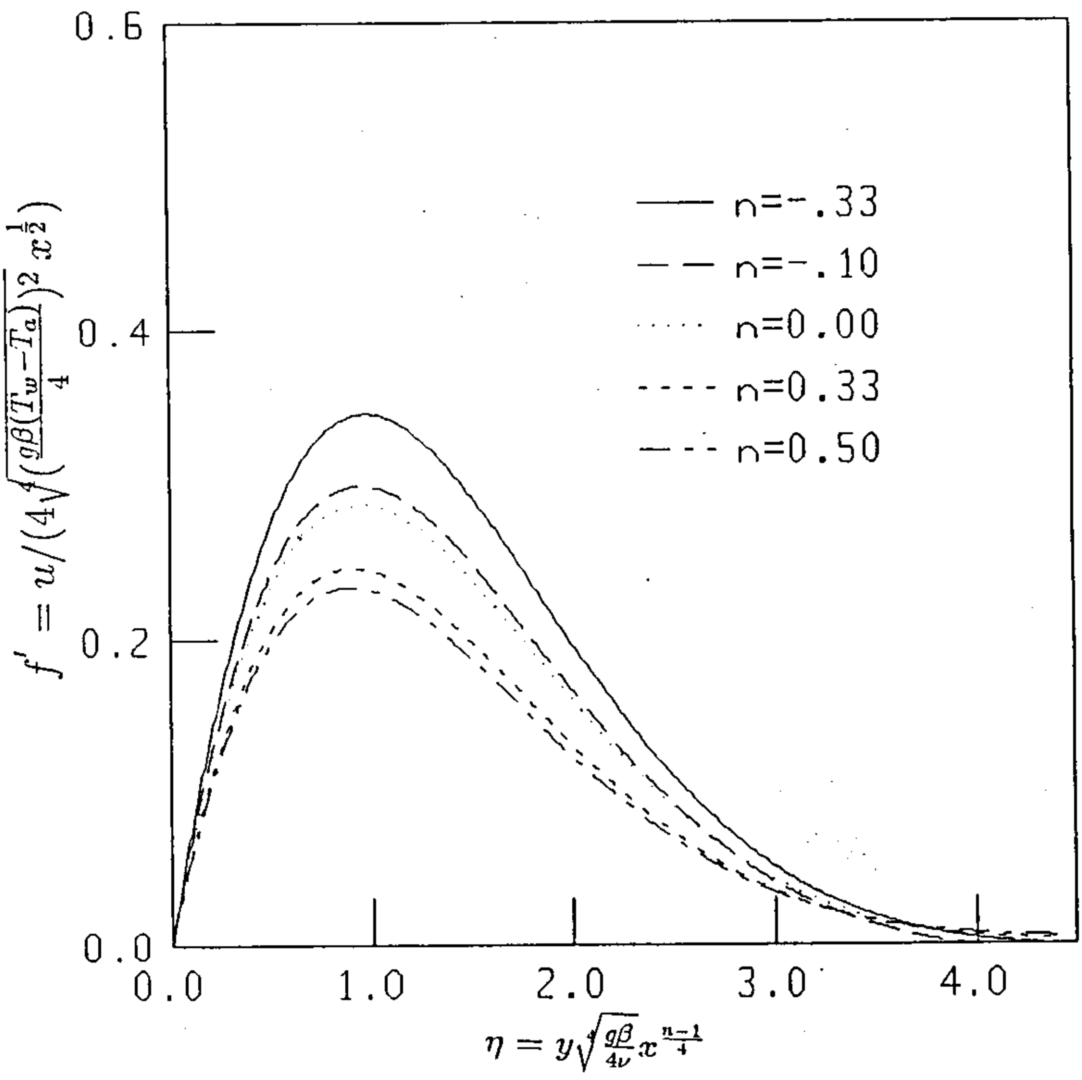


Fig.(4.7) The analytical velocity profiles

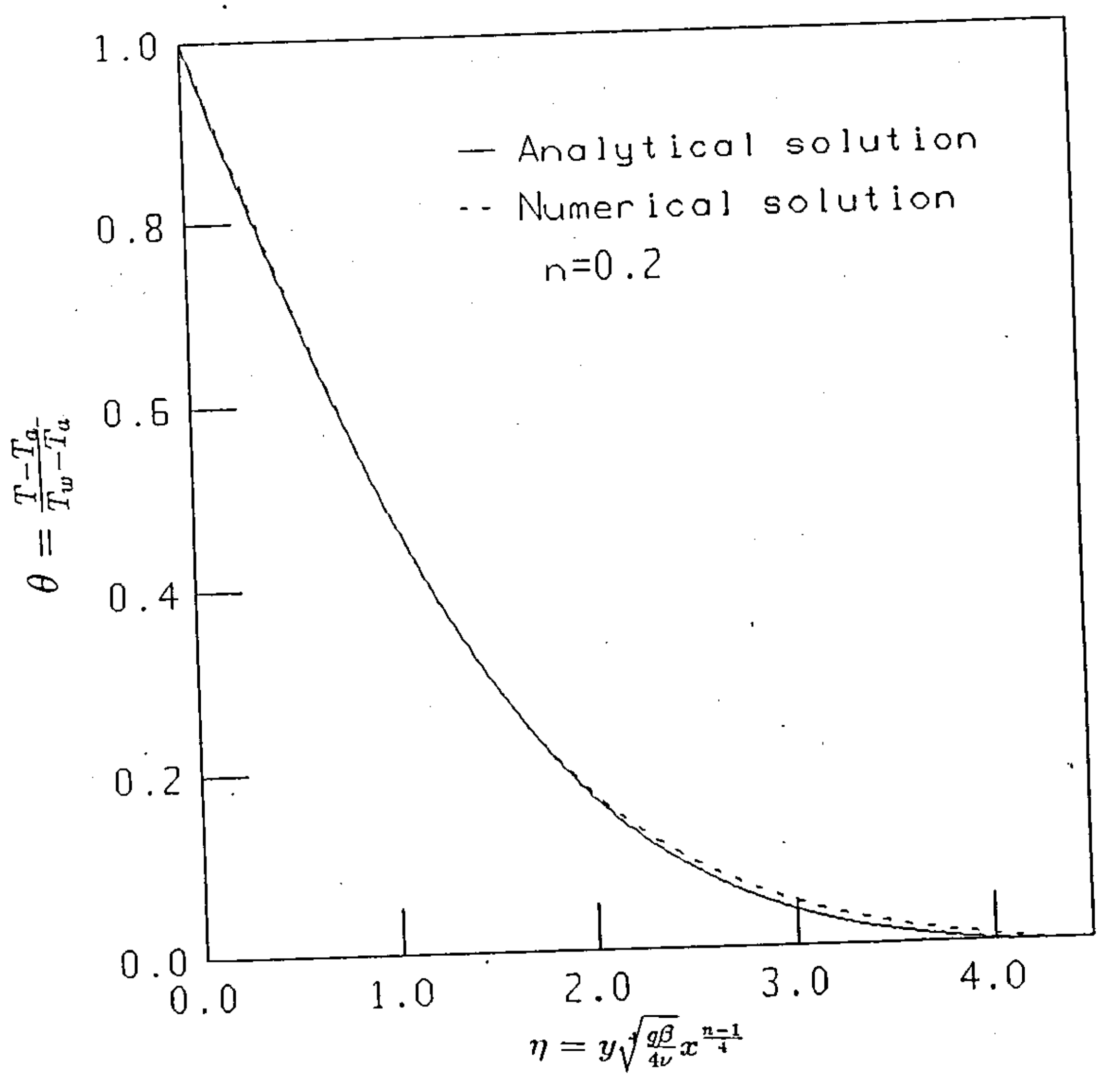


Fig. 4.8 Temperature distributions

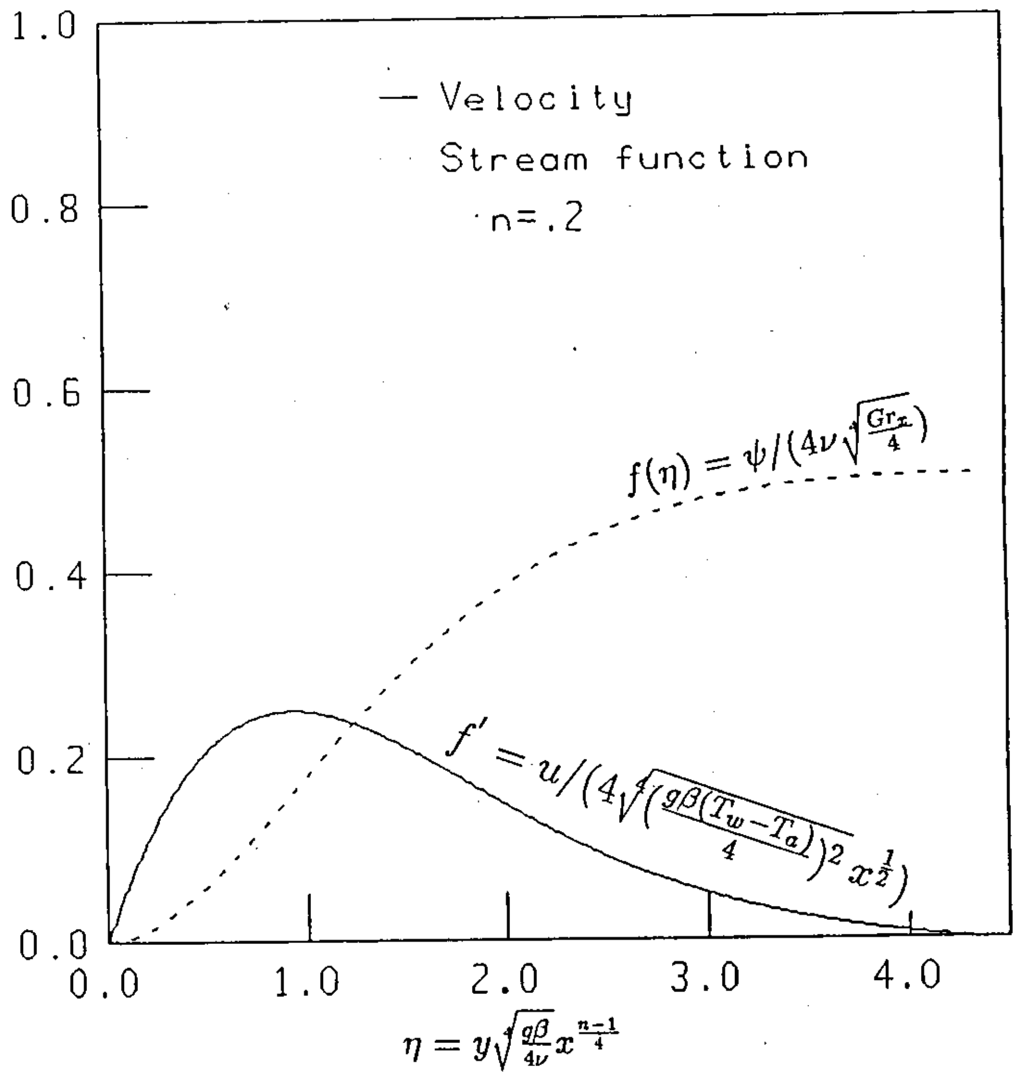


Fig.4.9 The analytical stream function and velocity profiles

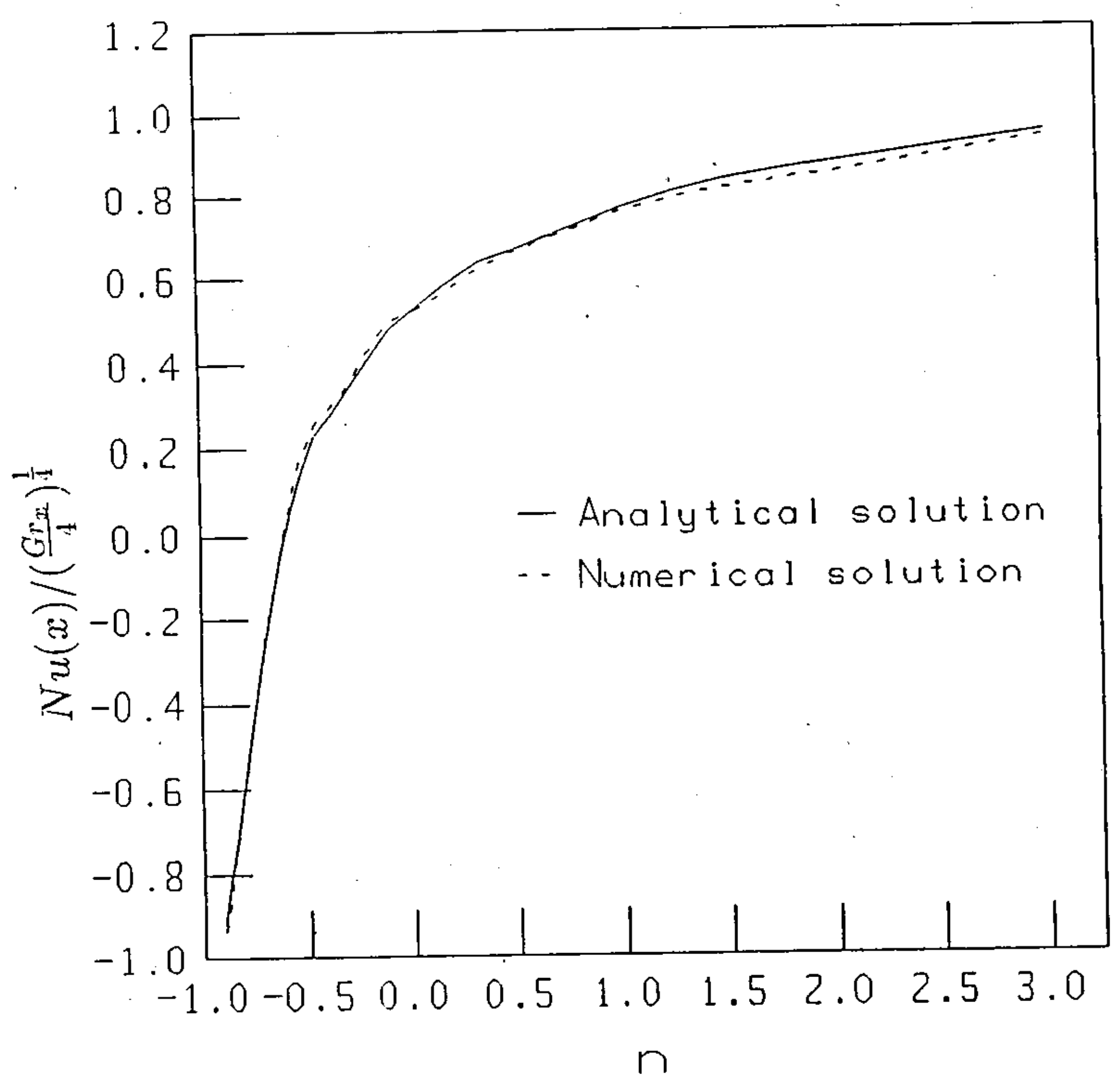


Fig.4.10 Dependence of the local Nusselt number on the value of n, for a power-law surface temperature distribution.

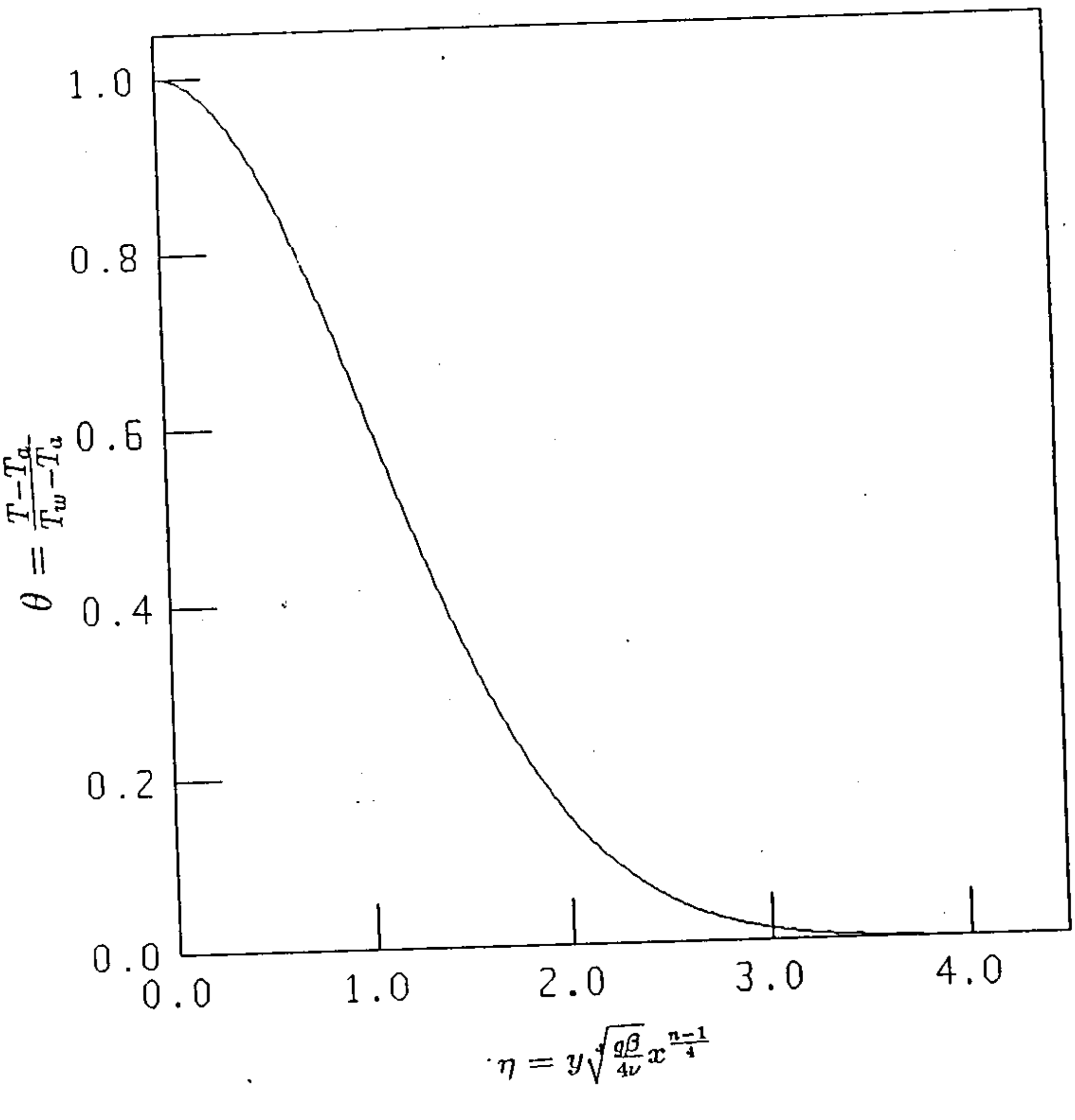


Fig. 4.11 analytical temperature distribution for n=-.6

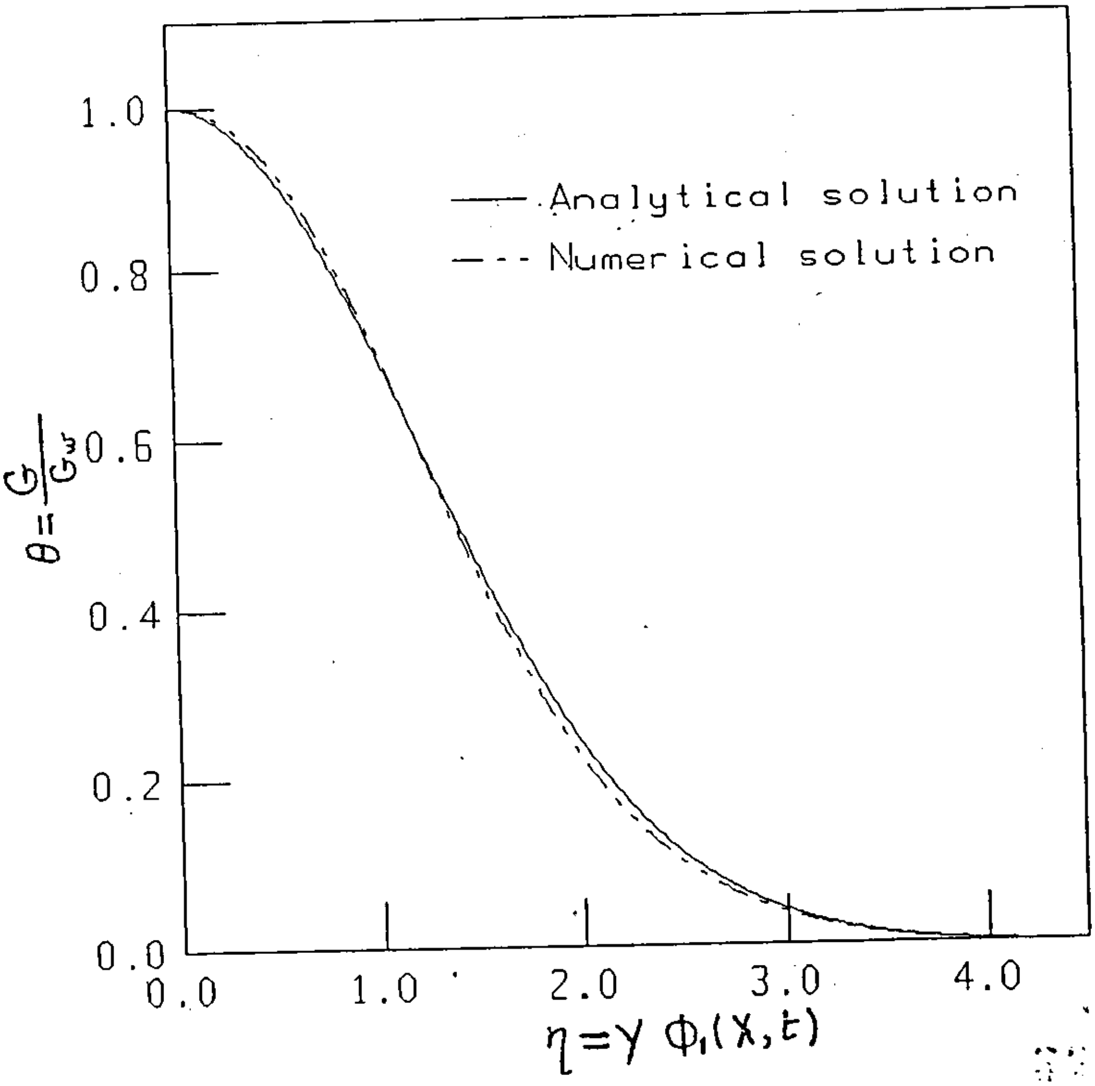


Fig.4.12 Temperature distributions for unsteady state conditions

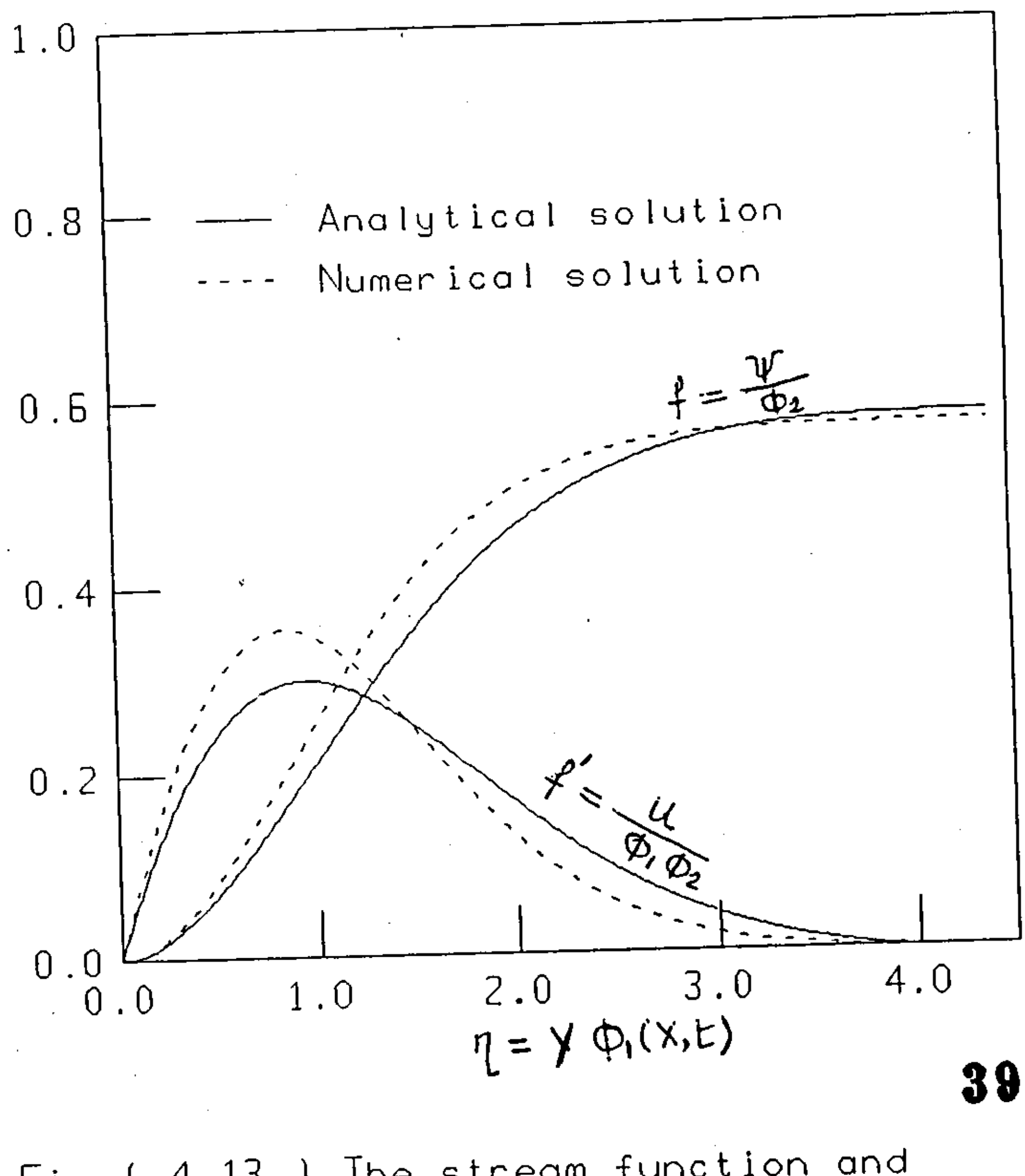


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Fig.(4.13) The stream function and velocity profiles for unsteady state case

ملخص رسالة الماجستير باللغة العربية

" طريقة جديدة تقريبية لحل مشكلة انتقال الحرارة في حائط ترمبو "

ان السمة الغالبة على هذا البحث هي السمة النظرية . لقد تم تطوير طريقة جديدة لايجاد حل تحليلي لمشكلة انتقال الحرارة في حائط ترمبو .

تمّ ايجاد حل تحليلي للمعادلات التي تصف عملية انتقال الحرارة داخل حائط ترمبو وذلك في حالة عدم التغيّر مع الزمن وعندما كانت درجة حرارة الحائط ثابته .

تم بحث تأثير الزمن على النموذج الحرارى وتوزيع السرعة داخل الحائط حيث تمّ ايجاد الحل التحليلي للمعادلات التي تصف هذه المشكلة .

أيضا تمّ ايجاد الحل التحليلي للمعادلات عندما تكون درجة حرارة الحائط ليست ثابتة وانما متغيرة حسب قانون قوى في (x) .

لقد تمت مقارنة النتائج التحليلية مع تلك التي حصل عليها بطرق عددية مختلفة ، وقد تمت المقارنة أيضا مع بعض النتائج العملية المتوفرة لدينا .

وقد أظهرت المقارنة موافقة جيدة بين النتائج التحليلية والعددية والعملية .